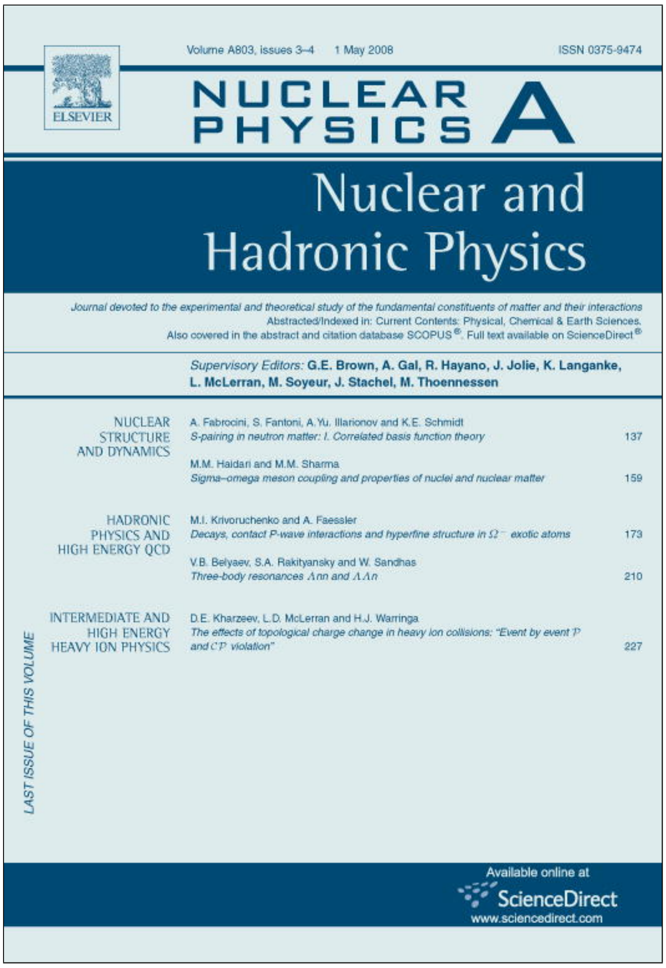

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Three-body resonances Λnn and $\Lambda\Lambda n$

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Abstract

Possible bound and resonant states of the hypernuclear systems Λnn and $\Lambda\Lambda n$ are sought as zeros of the corresponding three-body Jost functions calculated within the framework of the hyperspherical approach with local two-body S-wave potentials describing the nn, Λn, and $\Lambda\Lambda$ interactions. Very wide near-threshold resonance is found for the Λnn system. The position of this resonance turned out to be sensitive to the choice of the Λn-potential. Bound Λnn state only appears if the two-body potentials are multiplied by a factor of ~ 1.5. The system $\Lambda\Lambda n$ can only have a near-threshold resonance with the $\Lambda\Lambda$-potential which overbinds the hypernucleus ${}_{\Lambda\Lambda}^{6}\mathrm{He}$.

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Keywords: Lambda-nucleon potential; Hypernuclei; Three-body resonance; Hyperspherical harmonics; Jost function

1. Introduction

The Λ-hyperon belongs to a wide class of particles that are not in abundance in this world and therefore are not freely available for scattering experiments. The properties of their interaction with other particles are studied indirectly. For example, the most important and established way of studying the ΛN interaction consists in measuring and calculating the spectral properties of

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the so-called Λ-hypernuclei (see, for example, Refs. [1,2] and references therein), which are bound states of Λ-particles inside atomic nuclei. The most convenient for this purpose are very light nuclei with $A \lesssim 10$. Firstly, because such simple systems have simple spectra with only few well separated levels, and secondly, because they allow a reliable theoretical modelling based on rigorous few-body methods.

The hyperon–nucleon attraction is insufficient to bind a ΛN pair. The simplest hypernucleus is therefore the hypertriton ${}^3_\Lambda$H, i.e. a bound Λpn complex. Its binding energy is very small (the Λ particle is separated at ~ 0.15 MeV) [3–5]. So, it looks like a deuteron core surrounded by a Λ-halo [3,5].

Similarly to traditional (non-strange) nuclear physics, where the deuteron is the first testing ground for any NN-potential, the system ΛNN is used to constrain new models of the hyperon–nucleon interaction. This system was recently analyzed in Refs. [5,6] using rigorous three-body equations with the potentials constructed within the constituent quark model. The authors of Ref. [5] gave another proof that the coupling between the ΛNN and ΣNN channels is very important for the hypertriton binding and showed that ${}^3_\Lambda$H is the only bound state of the ΛNN system. Their comprehensive analysis lacks only one thing: they did not consider possible three-body resonances. Meanwhile their results give a strong indication that such resonances may exist and be located not far from the threshold energy. Indeed, they found that the channel Λnn is attractive but not sufficient to produce a bound state, and the curve for its Fredholm determinant turns towards zero near the threshold energy (see Fig. 4 of Ref. [5]). In our present paper, we partly fill in the gap by considering the Λnn resonance state.

The ΛN- and $\Lambda\Lambda$-potentials are usually constructed in such a way that the calculations with these potentials reproduce experimentally known bound states of the hypernuclei. Unfortunately, it is very difficult to do scattering experiments with the Λ-particles because of their short lifetime ($\sim 10^{-10}$ s) and extremely low intensity of the beams that can be obtained.

It is well known that even when scattering data are available in full, it is impossible to construct an interaction potential in a unique way. One can always obtain different but phase-equivalent potentials (see, for example, Ref. [7]). In this respect the ΛN-case is beyond any hope since only few experimental points for the Λp scattering are available [8,9]. During the decades of studying the hypernuclei many features of the ΛN-interaction have been revealed. However the comparison of the theoretical and experimental spectra remains inconclusive. Different potentials lead to almost the same spectra of the hypernuclei. We therefore need an additional tool for testing the potentials.

In principle, such a tool could be based on studying the Λ-nucleus resonances, if they do exist [10,11]. Indeed, while the scattering and bound states mostly reflect the on-shell properties of the interaction, the resonances strongly depend on its off-shell characteristics, which may be different for phase-equivalent potentials.

Our present work is an attempt to attract the attention of both theoreticians and experimentalists to the low-energy resonances in the Λ-nuclear systems. As an example, we consider the three-body systems Λnn and $\Lambda\Lambda n$ in the minimal approximation, $[L] = [L_{\min}]$, of the hyperspherical harmonics approach. By locating the S-matrix pole on the second (unphysical) sheet of the complex energy surface, we show that the system Λnn has a near-threshold resonant state. The position of the pole turns out to be strongly dependent on the choice of the ΛN-potential. This fact supports the idea that the studying of the Λ-nucleus resonances could be very important for finding an adequate ΛN-potential.

The demands for adequate hyperon–nucleon (YN) and hyperon–hyperon (YY) potentials come not only from nuclear physics itself, but also from astrophysics. The studies of the neutron stars show that these very dense and compact objects are in fact "giant hypernuclei" (see, for example, Ref. [12] and references therein). The Λ-particles appear inside neutron stars when the density becomes approximately two times higher than the ordinary nuclear density. The equation of state, describing a neutron star, involves all the inter-particle potentials and therefore its solutions depend on their properties. In particular, the strength of the short-range repulsion in the pairs ΛN and $\Lambda\Lambda$ is crucial for determining the maximum mass and size of a neutron star. The repulsive nature of the Λnn three-body force (if it is indeed repulsive) would lead to additional stability of neutron stars. Moreover, the two-body YY interactions regulate the cooling behaviour of massive neutron stars [12].

So, the studies of hypernuclear systems are not only important for reaching a better understanding of the physics of strange particles, but may also have an important impact on some other branches of science. This is why the research in this field is carried on by many theoretical groups and experimental laboratories.

2. Three-body Jost function

There are several different ways of locating quantum resonances. The most adequate are the methods based on the rigorous definition of resonances as the S-matrix poles at complex energies. This definition is universal and applicable to the systems involving more than just two colliding particles. Of course, the problem of locating the S-matrix poles is not an easy task, and especially for few-body systems. There are different approaches to this problem. To the best of our knowledge, so far only one of them has been applied to study the hyperon–nucleus resonant states. This was done in Ref. [10] using an analytic continuation of the rigorous three-body equations proposed by Alt, Grassberger, and Sandhas [13] and known as the AGS-equations. In our present paper, we follow a different approach based on direct calculation of the Jost function using the method suggested in Ref. [14].

The three-body systems we consider in the present paper, namely, Λnn and $\Lambda\Lambda n$, do not have bound states in any of the two-body subsystems nn, Λn, or $\Lambda\Lambda$. The only possible collision process for them is therefore the $3 \to 3$ scattering. The wave functions describing the systems that cannot form clusters behave asymptotically as linear combinations of the incoming and outgoing hyperspherical waves (see, for example, Ref. [15]). Thus it is convenient to describe the three-body configuration using the hyperspherical coordinates, among which only one (the hyperradius) runs from zero to infinity while all the others (the hyperangles) vary within finite ranges.

Within the hyperspherical approach, the wave function is expanded in an infinite series over the hyperspherical harmonics (similarly to the partial wave decomposition in the two-body problem), and we end up with an infinite system of coupled hyperradial equations, which is truncated in practical calculations. All the details of the hyperspherical approach can be found, for example, in the review by M. Fabre de la Ripelle [16].

It should be noted that although the two-body potentials and masses for the three-body systems Λnn and $\Lambda\Lambda n$ are different, they can be treated using exactly the same equations. Indeed, in both of these systems, we have two identical particles with spin $1/2$ and a third particle of the same spin. In what follows, we therefore consider a general system of this type.

Let m_1 be the mass of one of the identical particles, and m_2 be the mass of the third particle. Then the total mass of the system is $M = 2m_1 + m_2$ and the reduced masses for the identical pair and for the third particle are $\mu_1 = m_1/2$ and $\mu_2 = 2m_1 m_2/M$, respectively. With the Jacobi

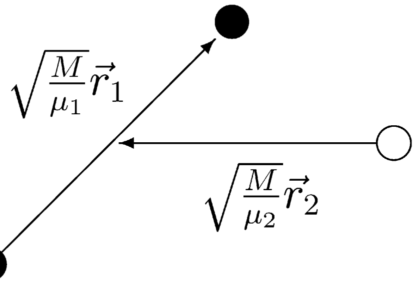

Fig. 1. Jacobi vectors defining the spatial configuration of a three-body system of two identical (filled circles) and one different (open circle) particles.

coordinates shown in Fig. 1, the three-body Schrödinger equation can be written as

$$\left(\partial_r^2 + \frac{5}{r}\partial_r - \frac{1}{r^2}\mathcal{L}^2 + k^2 - V\right)\Psi^{[s]}_{\vec{k}_1,\vec{k}_2}(\vec{r}_1, \vec{r}_2) = 0, \tag{1}$$

where

$$V = 2M(U_{12} + U_{13} + U_{23}) \tag{2}$$

is the sum of the two-body potentials U_{ij}, the vectors $\{\vec{k}_1, \vec{k}_2\}$ represent the incident momenta of the three-body collision along the corresponding configuration vectors $\{\vec{r}_1, \vec{r}_2\}$, the superscript $[s] = ((s_1 s_2)s_{12}s_3)s\sigma$ denotes the spin quantum numbers for the spin-addition scheme $\vec{s} = (\vec{s}_1 + \vec{s}_2) + \vec{s}_3$, the variable

$$r = \sqrt{r_1^2 + r_2^2} \tag{3}$$

is the hyperradius that gives the "collective" size of the system, k is related to the total energy, $k^2 = 2ME$, and can be called the hypermomentum, and the operator $\mathcal{L}^2$ absorbs all the angular variables. It is defined as

$$\mathcal{L}^2 = -\frac{\partial^2}{\partial\alpha^2} - 4\cot(2\alpha)\frac{\partial}{\partial\alpha} + \frac{1}{\cos^2\alpha}\vec{\ell}^{\,2}_{\vec{r}_1} + \frac{1}{\sin^2\alpha}\vec{\ell}^{\,2}_{\vec{r}_2} \tag{4}$$

with $\alpha = \arctan(r_2/r_1)$, $0 \leqslant \alpha \leqslant \pi/2$, and $\vec{\ell}_{\vec{r}_i}$ being the operators of the angular momenta associated with the corresponding Jacobi coordinates. The solutions of the eigenvalue problem

$$\mathcal{L}^2 Y_{[L]}(\omega) = L(L+4)Y_{[L]}(\omega) \tag{5}$$

are the so-called hyperspherical harmonics that depend on the hyperangles $\omega = \{\Omega_{\vec{r}_1}, \Omega_{\vec{r}_2}, \alpha\}$ including the spherical angles $\Omega_{\vec{r}_i}$ of the vectors $\vec{r}_i$ and the angle α that determines the ratio r_2/r_1. The subscript $[L]$ is the multi-index $[L] = \{L, \ell_1, \ell_2, \ell, m\}$ that includes the grand orbital quantum number,

$$L = \ell_1 + \ell_2 + 2n, \quad n = 0, 1, 2, \ldots, \tag{6}$$

as well as the angular momenta associated with the Jacobi vectors and the total angular momentum ℓ together with its third component m. Combining $Y_{[L]}(\omega)$ with the spin states $\chi_{[s]} = |((s_1 s_2)s_{12}s_3)s\sigma\rangle$, we obtain the functions

$$\Phi^{jj_z}_{[L]}(\omega) = \sum_{m\sigma}\langle \ell m s\sigma | j j_z\rangle Y_{[L]}(\omega)\chi_{[s]} \tag{7}$$

that constitute a full ortho-normal set of states with a given total angular momentum j in the spin-angular subspace.

Similarly to the two-body partial wave decomposition, we can expand a solution of Eq. (1) in the infinite series over the hyperspherical harmonics,

$$\Psi^{[s]}_{\vec{k}_1,\vec{k}_2}(\vec{r}_1,\vec{r}_2) = \frac{1}{r^{5/2}} \sum_{[L][L']jj_z} u^{jj_z}_{[L][L']}(E,r)\Phi^{jj_z}_{[L]}(\omega_{\vec{r}})\Phi^{jj_z*}_{[L']}(\omega_{\vec{k}}), \tag{8}$$

where the hyperangle sets $\omega_{\vec{r}}$ and $\omega_{\vec{k}}$ are associated with the pairs $\{\vec{r}_1,\vec{r}_2\}$ and $\{\vec{k}_1,\vec{k}_2\}$, respectively. After substituting this expansion into Eq. (1) and doing the projection onto the functions $\Phi^{jj_z}_{[L]}$, we end up with the following system of hyperradial equations

$$\left[\partial_r^2 + k^2 - \frac{\lambda(\lambda+1)}{r^2}\right]u_{[L][L']} = \sum_{[L'']} V_{[L][L'']}u_{[L''][L']}, \tag{9}$$

where for the sake of simplicity we dropped the superscripts jj_z (indicating the conserving total angular momentum). In Eq. (9),

$$V_{[L][L']}(r) = 2M \int \Phi^{jj_z*}_{[L]}(\omega)(U_{12}+U_{13}+U_{23})\Phi^{jj_z}_{[L']}(\omega)\,d\omega, \tag{10}$$

and $\lambda = L + 3/2$. Since we consider a system that cannot form clusters, the asymptotic behaviour of its wave function may only involve the incoming and outgoing hyperspherical waves $\sim \exp(\mp ikr)$. We therefore look for the solution of matrix equation (9) as

$$u_{[L][L']}(E,r) = h^{(-)}_\lambda(kr)F^{(\mathrm{in})}_{[L][L']}(E,r) + h^{(+)}_\lambda(kr)F^{(\mathrm{out})}_{[L][L']}(E,r), \tag{11}$$

where the incoming and outgoing hyperspherical waves described by the Riccati–Hankel functions,

$$h^{(\pm)}_\lambda(kr) \underset{|kr|\to\infty}{\longrightarrow} \mp i \exp\big[\pm i(kr - \lambda\pi/2)\big], \tag{12}$$

are included explicitly. The matrices $F^{(\mathrm{in/out})}_{[L][L']}(E,r)$ are new unknown functions. In the theory of ordinary differential equations, this way of finding solution is known as the variation parameters method (see, for example, Ref. [17]).

Since instead of one unknown matrix $u_{[L][L']}$ we introduce two unknown matrices $F^{(\mathrm{in/out})}_{[L][L']}$, they cannot be independent. We therefore can impose an arbitrary condition that relates them to each other. As such condition, it is convenient to choose the following equation

$$h^{(-)}_\lambda(kr)\partial_r F^{(\mathrm{in})}_{[L][L']}(E,r) + h^{(+)}_\lambda(kr)\partial_r F^{(\mathrm{out})}_{[L][L']}(E,r) = 0, \tag{13}$$

which is standard in the variation parameters method and is called the Lagrange condition. Substituting the ansatz (11) into the hyperradial equation (9) and using the condition (13), we obtain the following system of first order equations for these unknown matrices

$$\begin{cases} \partial_r F^{(\mathrm{in})}_{[L][L']} = -\frac{h^{(+)}_\lambda}{2ik}\sum_{[L'']} V_{[L][L'']}[h^{(-)}_{\lambda''}F^{(\mathrm{in})}_{[L''][L']} + h^{(+)}_{\lambda''}F^{(\mathrm{out})}_{[L''][L']}], \\ \partial_r F^{(\mathrm{out})}_{[L][L']} = +\frac{h^{(-)}_\lambda}{2ik}\sum_{[L'']} V_{[L][L'']}[h^{(-)}_{\lambda''}F^{(\mathrm{in})}_{[L''][L']} + h^{(+)}_{\lambda''}F^{(\mathrm{out})}_{[L''][L']}], \end{cases} \tag{14}$$

which are equivalent to the second order Eq. (9). The regularity of a physical wave function at $r = 0$ implies the following boundary conditions

$$F^{(\mathrm{in})}_{[L][L']}(E,0) = F^{(\mathrm{out})}_{[L][L']}(E,0) = \delta_{[L][L']}. \tag{15}$$

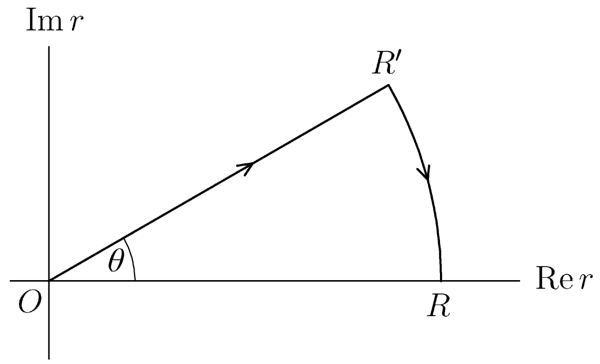

Fig. 2. Deformed contour for integrating Eqs. (14) from $r = 0$ to $r = R$ when the energy is complex.

With these conditions, the columns of the matrix $u_{[L][L']}(E,r)$ are not only regular but linearly independent as well. Therefore any regular column $\phi_{[L]}(E,r)$ obeying Eq. (9), can be written as a linear combination of the columns of matrix $u_{[L][L']}(E,r)$. In other words, the matrix $u_{[L][L']}(E,r)$ is a complete basis for the regular solutions.

At large hyperradius where the potentials vanish, i.e.

$$V_{[L][L']}(r) \underset{r\to\infty}{\longrightarrow} 0, \tag{16}$$

the right-hand sides of Eqs. (14) should tend to zero and therefore the matrices $F^{(\text{in/out})}_{[L][L']}(E,r)$ converge to the energy-dependent constants,

$$f^{(\text{in/out})}_{[L][L']}(E) = \lim_{r\to\infty} F^{(\text{in/out})}_{[L][L']}(E,r), \tag{17}$$

that by analogy with the two-body case can be called the Jost matrices. The convergency of these limits, however, depends on the choice of the energy E and on how fast the potential matrix $V_{[L][L']}(r)$ vanishes when $r \to \infty$.

When the energy is real and positive (scattering states), the vanishing of the right-hand sides of Eqs. (14) at large distances is completely determined by the behaviour of $V_{[L][L']}(r)$. It can be shown that in such a case the limits (17) exist if $V_{[L][L']}(r)$ vanishes faster than $1/r$.

With negative and complex energies there is a technical complication. The problem is that one of the Riccati–Hankel functions on the right-hand side of Eqs. (14) is always exponentially diverging. Therefore, if at large distances the potential matrix vanishes not fast enough, the convergency of (17) is not achieved. This problem can be easily circumvented by using different path to the far-away point (see Fig. 2).

This is known as the complex rotation of the coordinate. All the details concerning convergency of the limits (17) and the use of complex rotation for this purpose can be found in Refs. [14,18–22].

As was said before, the columns of the matrix function $u_{[L][L']}(E,r)$ constitute the regular basis using which we can construct a physical solution $\phi_{[L]}(E,r)$ with given boundary conditions at infinity,

$$\phi_{[L]}(E,r) = \sum_{[L']} u_{[L][L']}(E,r) C_{[L']}, \tag{18}$$

where $C_{[L]}$ are the combination coefficients.

The spectral points $E_{\rm n}$ (bound and resonant states) are those at which the physical solution has only outgoing waves in its asymptotics, i.e. when

$$\sum_{[L']} f^{(\mathrm{in})}_{[L][L']}(E_\mathrm{n}) C_{[L']} = 0. \tag{19}$$

This homogeneous system has a non-trivial solution if and only if

$$\det f^{(\mathrm{in})}_{[L][L']}(E_\mathrm{n}) = 0, \tag{20}$$

which determines the spectral energies E_n. As can be easily shown [23], the S-matrix is given by

$$S(E) = f^{(\mathrm{out})}_\ell(E)\big[f^{(\mathrm{in})}_\ell(E)\big]^{-1} \tag{21}$$

and therefore at the energies E_n it has poles.

3. Two-body potentials

For our calculations, we need two-body potentials describing the interaction between two neutrons, Λ and neutron, and between two Λ-particles. As is shown in Ref. [24], for all these potentials the same functional form can be used, namely,

$$U(\rho) = \left[A_1(\rho) - \frac{1+P^\sigma}{2}A_2(\rho) - \frac{1-P^\sigma}{2}A_3(\rho)\right]\left[\frac{\beta}{2} + \frac{1}{2}(2-\beta)P^r\right], \tag{22}$$

$$A_n(\rho) = W_n \exp\big(-a_n\rho^2\big), \quad n = 1, 2, 3, \tag{23}$$

where P^σ and P^r are the permutation operators in the spin and configuration spaces, respectively. The sets of the corresponding parameters are given in Table 1. In order to explore how sensitive the positions of the three-body resonances are to the choice of underlying two-body potentials, we did the calculations with three different sets of parameters for the Λn-potential.

The $\Lambda\Lambda$-potential (22) of Ref. [24] overbinds the hypernucleus ${}^{6}_{\Lambda\Lambda}\mathrm{He}$. For the energy of $\Lambda\Lambda$-separation from this hypernucleus (calculated in the same paper) it gives $B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) =$ 15.1 MeV, and for the incremental binding energy $\Delta B_{\Lambda\Lambda}$, defined as

$$\Delta B_{\Lambda\Lambda}\big({}^{6}_{\Lambda\Lambda}\mathrm{He}\big) = B_{\Lambda\Lambda}\big({}^{6}_{\Lambda\Lambda}\mathrm{He}\big) - 2B_\Lambda\big({}^{5}_{\Lambda}\mathrm{He}\big), \tag{24}$$

the authors of Ref. [24] obtain $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 5.2$ MeV. Meanwhile the latest experimental values for these quantities are [25]

$$B_{\Lambda\Lambda}\big({}^{6}_{\Lambda\Lambda}\mathrm{He}\big) = 7.25 \pm 0.19\ \mathrm{MeV}, \tag{25}$$

$$\Delta B_{\Lambda\Lambda}\big({}^{6}_{\Lambda\Lambda}\mathrm{He}\big) = 1.01 \pm 0.20\ \mathrm{MeV}. \tag{26}$$

Table 1
Parameters of the potential (22) for the pairs nn, $\Lambda\Lambda$, and Λn. For the system Λn, three different sets of parameters (denoted as A, B, and C) are given. All the parameters are taken from Ref. [24]

	nn	$\Lambda\Lambda$	Λn (A)	Λn (B)	Λn (C)
W_1 (MeV)	200.0	200.0	200.0	600.0	5000
W_2 (MeV)	178.0	0	106.5	52.61	47.87
W_3 (MeV)	91.85	130.8	118.65	66.22	61.66
a_1 (fm^{-2})	1.487	2.776	1.638	5.824	18.04
a_2 (fm^{-2})	0.639	0	0.7864	0.6582	0.6399
a_3 (fm^{-2})	0.465	1.062	0.7513	0.6460	0.6325
β	1	1	1.5	1.5	1.5

Table 2
Parameters of $\Lambda\Lambda$-potential (27). The additional factor $\gamma = 0.6598$ in the second line was introduced in Ref. [26] in order to precisely reproduce the experimental value (26)

n	W_n (MeV)	b_n (fm)
1	-21.49	1342
2	$-379.1 \cdot \gamma$	0777
3	9324	0350

Table 3
Parameters of the effective-range expansion defined by Eq. (28), for the potentials used in our calculations

Potential	Spin state	a (fm)	r_0 (fm)	V_0 (fm^3)
nn	$^1S_0$	-16.904	2.884	0.687
Λn (A)	$^1S_0$	-2.260	3.212	0.637
Λn (B)	$^1S_0$	-2.530	3.077	0.577
Λn (C)	$^1S_0$	-2.529	3.078	0.576
Λn (A)	$^3S_1$	-1.057	4.616	1.206
Λn (B)	$^3S_1$	-1.199	4.253	1.022
Λn (C)	$^3S_1$	-1.199	4.253	1.017
$\Lambda\Lambda$, Ref. [24]	$^1S_0$	-3.011	2.159	0.216
$\Lambda\Lambda$, Ref. [26]	$^1S_0$	-0.778	6.564	2.640

In Ref. [26] another $\Lambda\Lambda$-potential is used. It has a weaker attraction and therefore enables the authors to reproduce the experimental values (25), (26) in their rigorous few-body calculations. Similarly to (22), the $\Lambda\Lambda$-potential of Ref. [26] also consists of three Gaussian terms,

$$U_{\Lambda\Lambda}(\rho) = \sum_{n=1}^{3} W_n \exp\left(-\frac{\rho^2}{b_n^2}\right). \tag{27}$$

The parameters of this potential for the state $^1S_0$ are given in Table 2. In our calculations, we used both $\Lambda\Lambda$-potentials, namely, from Refs. [24] and [26]. The near-threshold characteristics of all two-body potentials used in the present paper are given in Table 3. They include the scattering length a, effective radius r_0, and scattering volume V_0 defined by the following low-energy expansion

$$k\cot\delta_0 = -\frac{1}{a} + \frac{r_0}{2}k^2 + V_0 k^4 + \cdots. \tag{28}$$

For each two-body potential, these quantities were determined using the method described in Appendix A.

4. The minimal approximation

The system (14) consists of infinite number of equations. For any practical calculation, one has to truncate it somewhere. Before going any further, it is very logical to try the simplest approximation, namely, when only the first terms of the sums on the right-hand sides of Eqs. (14) are retained. This corresponds to the minimal ($n = 0$) value of the grand orbital number (6) and is

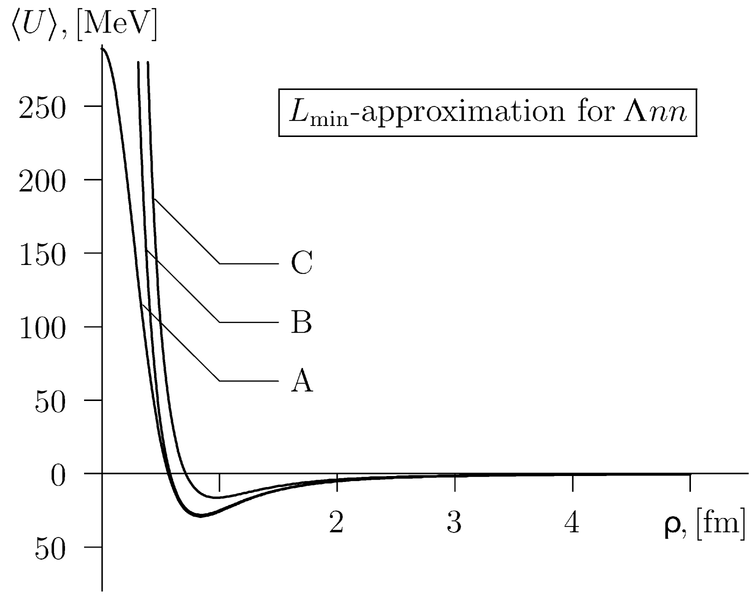

Fig. 3. The hypercentral potential defined by Eq. (30) for the system Λnn with the three choices (A, B, and C) of the Λn interaction.

called the hypercentral approximation, $[L] = [L_{\min}]$. We assume that the two-body subsystems are in the S-wave states ($\ell_1 = \ell_2 = 0$), which means that

$$\lambda = \lambda_{\min} = \frac{3}{2}.$$

So, in the minimal approximation, instead of the infinite system (14), we remain with only one equation,

$$\left[\partial_r^2 + k^2 - \frac{\lambda_{\min}(\lambda_{\min}+1)}{r^2}\right]u(E,r) = 2M\langle U\rangle u(E,r), \tag{29}$$

where all unnecessary subscripts are dropped, and the brackets on the right-hand side mean the following integration

$$\langle U\rangle(r) = \int \Phi^{jj_z*}_{[L_{\min}]}(\omega)(U_{12}+U_{13}+U_{23})\Phi^{jj_z}_{[L_{\min}]}(\omega)\,d\omega. \tag{30}$$

From the mathematical point of view, Eq. (29) looks exactly like the two-body radial Schrödinger equation. The only difference is that the angular momentum is not an integer number.

The explicit expression for the integral (30) is given in Appendix B. The hypercentral potentials $\langle U\rangle$ for the systems Λnn and $\Lambda\Lambda n$ are shown in Figs. 3 and 4. With these hyperradial potentials the corresponding differential equations determining the three-body Jost functions, were numerically solved with complex values of the energy. The results of these calculations are discussed next.

5. Numerical results and conclusion

When looking for zeros of the three-body Jost functions, we found that there were no such zeros at real negative energies. In other words, neither the system Λnn nor $\Lambda\Lambda n$ have bound states.

For the system Λnn, the only zero we found was located on the unphysical sheet of the energy surface, in the resonance domain. The energy of this resonance (for the three choices of

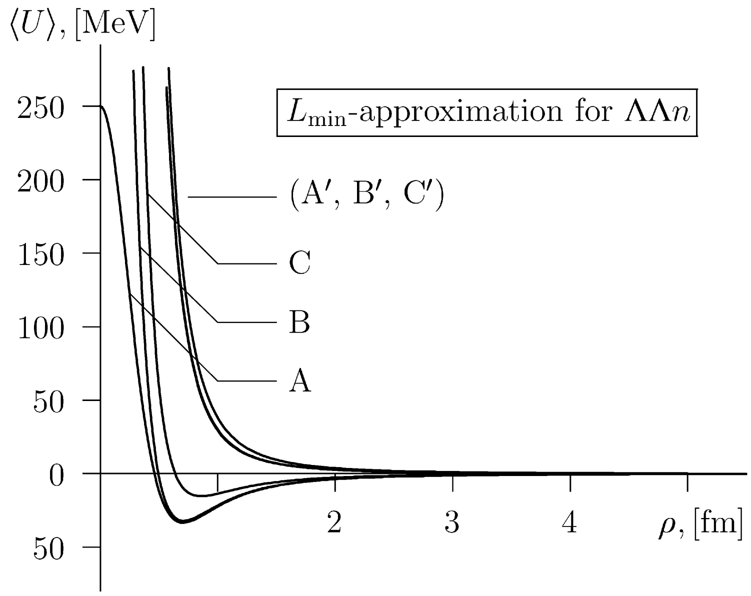

Fig. 4. The hypercentral potential defined by Eq. (30) for the system $\Lambda\Lambda n$. The curves marked as A, B, and C, are obtained with the $\Lambda\Lambda$-potential taken from Ref. [24] and with three different Λn-potentials (A, B, and C) taken from the same paper. The three almost identical curves jointly marked as (A′, B′, C′), are obtained with the $\Lambda\Lambda$-potential taken from Ref. [26] and the same three Λn-potentials from Ref. [24].

Table 4
Complex resonance energies $E_0 = E_r - \frac{i}{2}\Gamma$ for the system Λnn with the three choices of Λn-potential

Λn-potential	A	B	C
E_0 (MeV)	$0.551 - \frac{i}{2}4.698$	$0.456 - \frac{i}{2}4.885$	$-0.149 - \frac{i}{2}5.783$

Table 5
Complex resonance energies $E_0 = E_r - \frac{i}{2}\Gamma$ for the system $\Lambda\Lambda n$ with the three choices of Λn-potential (see Table 1) and with the $\Lambda\Lambda$-potential of Ref. [24] that overbinds the hypernucleus ${}_{\Lambda\Lambda}^{6}$He

Λn-potential	A	B	C
E_0 (MeV)	$0.096 - \frac{i}{2}8.392$	$0.034 - \frac{i}{2}8.438$	$-0.552 - \frac{i}{2}8.681$

the Λn-potential) is given in Table 4 and shown in Fig. 5. As is seen, the position of the resonance strongly depends on the choice of the Λn-potential. For the choice "C", the resonance becomes sub-threshold.

In the case of $\Lambda\Lambda n$ system, the choice of the $\Lambda\Lambda$-potential turned out to be crucial. With the $\Lambda\Lambda$-potential (22) of Ref. [24], the hypercentral potential has an attractive part with all three choices (A, B, and C) of the Λn interaction (see Fig. 4). This attraction is however rather weak and as a result the corresponding resonance poles are located very close to the threshold. For the case "C", it is even below threshold. The corresponding resonance energies are given in Table 5 and shown in Fig. 5.

As already mentioned in Section 3, the potential of Ref. [24] overestimates the $\Lambda\Lambda$ attraction and thus overbinds the hypernucleus ${}_{\Lambda\Lambda}^{6}$He. When we replaced it with a more realistic potential (27) of Ref. [26], the $\Lambda\Lambda n$ hypercentral potential became pure repulsive, independent of the

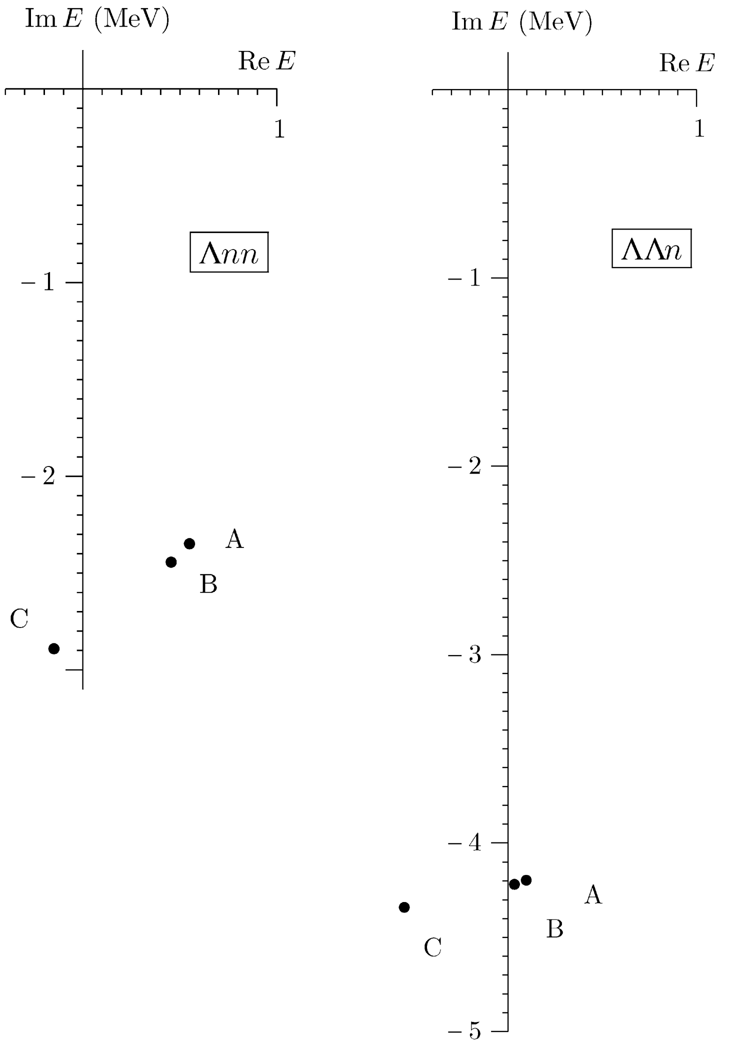

Fig. 5. Resonance points for the systems Λnn and $\Lambda\Lambda n$ found on the unphysical sheet of the energy surface with the three sets (A, B, and C) of parameters of the Λn-potential given in Table 1, and with the $\Lambda\Lambda$-potential of Ref. [24], that overbinds the hypernucleus ${}^{6}_{\Lambda\Lambda}\mathrm{He}$.

choice (A, B, or C) of the Λn-potential (see Fig. 4). Of course, the corresponding Jost function does not have zeros neither in the bound-state nor in the resonance domains.

In order to estimate how far our three-body system Λnn is from being bound, we artificially increased the depths of the potentials by multiplying them by a scaling factor. When this factor was increased from 1 upwards, the Jost function zeros moved towards the origin of the energy surface. At the value of approximately 1.5, the zeros crossed the threshold and moved onto the real negative axis. In other words, the bound state can appear if the potential strength is increased by $\sim 50\%$.

The fact that we did not find bound Λnn or $\Lambda\Lambda n$ states is not surprising at all. As is shown in Refs. [3,5], the system ΛNN in the state with the three-body isospin 1 and spin $s = 1/2$ is not bound even when the virtual processes of Λ–Σ conversion are taken into account, although this conversion increases the attraction in the system. Simple but convincing argumentation of Ref. [27] leads us to the conclusion that the $\Lambda\Lambda n$ system also cannot be bound. Indeed, the system $\Lambda\Lambda n$ is a "mirror" image of Λnn, where the Λ and n replace each other. This means that the potential term $U = U_{nn} + U_{\Lambda n} + U_{\Lambda n}$ of the three-body Hamiltonian is replaced with $U = U_{\Lambda\Lambda} + U_{\Lambda n} + U_{\Lambda n}$. Since the attraction of $U_{\Lambda\Lambda}$ is weaker than that of U_{nn}, we may conclude that the system $\Lambda\Lambda n$ has less chances to be bound than the system Λnn. The calculations performed in Refs. [24,28–30], show that even the heavier hypernucleus ${}^{4}_{\Lambda\Lambda}$H (i.e. the system $\Lambda\Lambda pn$) is bound very weakly, if bound at all.

Multiplying the two-body potentials by an appropriate scaling factor, we can always generate an artificial three-body bound state, i.e. a pole of the S-matrix on the physical sheet of the E-surface at a negative energy. Apparently this pole cannot disappear when the scaling factor returns to its natural value of 1. The pole simply moves via the threshold onto the unphysical sheet. Since the system Λnn is not far from being bound, the corresponding pole cannot be far away from the threshold energy. And indeed we located it at low energies.

What we found is, of course, an estimate. But it clearly shows that there is a near-threshold resonance of the systems Λnn. Actual location of the pole most probably is more close to the threshold energy. An inclusion of the channel ΛN–ΣN would definitely increase the attraction in our system (see Ref. [31]) and this would make the widths of the resonances smaller.

As we have demonstrated, the position of the resonance strongly depends on the choice of the two-body potentials. If such resonance is observed experimentally, it may serve as an additional instrument for constructing adequate YN- and YY-potentials. There are many possible reactions where the three-body resonance Λnn may manifest itself. As an example, we can mention the inelastic collision of the K^- meson with the α particle,

$$K^- + {}^4\mathrm{He} \longrightarrow p + \Lambda + n + n, \tag{31}$$

that produces a proton and the system we are looking for. If a short-lived cluster Λnn is formed in the final state of this collision, it should be seen in the corresponding two-body kinematics p–Λnn. The processes of the type (31) fall under the experimental programme AMADEUS [32] (in the INFN, Italy) and, in principle, this reaction could be thoroughly studied.

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Appendix A. Effective-range expansion

The scattering length a, effective radius r_0, scattering volume V_0, and constant factors of any further terms of the expansion (28) can be accurately calculated using a semianalytical representation of the wave function similar to Eq. (11). Let $u(k,r)$ be the radial wave function obeying the S-wave two-body Schrödinger equation,

$$\left(\partial_r^2 + k^2\right)u(k,r) = V(r)u(k,r). \tag{A.1}$$

Far away from the origin the right-hand side of this equation vanishes and u becomes a linear combination of $\sin(kr)$ and $\cos(kr)$. This suggests the following semianalytical ansatz for $u(k,r)$:

$$u(k,r) \equiv \sin(kr)A(k,r) + \cos(kr)B(k,r). \tag{A.2}$$

Since one unknown function u is replaced with two unknown functions A and B, they cannot be independent. We therefore can introduce an arbitrary relation between them. The most convenient is the condition

$$\sin(kr)\partial_r A(k,r) + \cos(kr)\partial_r B(k,r) \equiv 0, \tag{A.3}$$

which is known in the theory of differential equations as the Lagrange condition of the variation parameters method. Substituting the ansatz (A.2) into Eq. (A.1) and using the relation (A.3), we obtain the following equations for the new unknown functions:

$$\begin{cases} \partial_r A = \frac{\cos(kr)}{k}[\sin(kr)A + \cos(kr)B], \\ \partial_r B = -\frac{\sin(kr)}{k}[\sin(kr)A + \cos(kr)B], \end{cases} \tag{A.4}$$

with the boundary conditions $A(k,0) = 1/k$ and $B(k,0) = 0$, which follow from the condition of regularity, namely, $u(k,r) \to r$ when $r \to 0$.

Writing the Taylor series for the functions $\sin(kr)$ and $\cos(kr)$ in the form

$$\sin kr = k\sum_{n=0}^{\infty} k^{2n}\gamma_n(r), \tag{A.5}$$

$$\cos kr = \sum_{n=0}^{\infty} k^{2n}\eta_n(r), \tag{A.6}$$

where

$$\gamma_0 = r, \quad \gamma_1 = -\frac{r^3}{6}, \quad \gamma_2 = \frac{r^5}{120}, \quad \ldots$$

and

$$\eta_0 = 1, \quad \eta_1 = -\frac{r^2}{2}, \quad \eta_2 = \frac{r^4}{24}, \quad \ldots,$$

we see that the functions $A(k,r)$ and $B(k,r)$ can also be represented by similar series, namely,

$$A(k,r) = k^{-1}\sum_{n=0}^{\infty} k^{2n}\alpha_n(r), \tag{A.7}$$

$$B(k,r) = \sum_{n=0}^{\infty} k^{2n}\beta_n(r). \tag{A.8}$$

Substituting the expansions (A.5), (A.6), (A.7), (A.8) into the system (A.4) and comparing the terms on its left- and right-hand sides, we obtain differential equations for the k-independent coefficients of the expansions (A.7), (A.8),

$$\begin{cases} \alpha_n' = \sum_{i+j+k=n} \eta_i V(\gamma_j\alpha_k + \eta_j\beta_k), \\ \beta_n' = -\sum_{i+j+k=n} \gamma_i V(\gamma_j\alpha_k + \eta_j\beta_k), \end{cases} \tag{A.9}$$

with simple boundary conditions $\alpha_n(0) = \delta_{n0}$, $\beta_n(0) = 0$ for all n.

The remarkable fact is that, thanks to the condition $i + j + k = n$, not all equations in the system (A.9) are coupled to each other. Indeed, for $n = 0$ we have

$$\begin{cases} \alpha_0' = \eta_0 V(\gamma_0\alpha_0 + \eta_0\beta_0), \\ \beta_0' = -\gamma_0 V(\gamma_0\alpha_0 + \eta_0\beta_0). \end{cases} \tag{A.10}$$

The equations for $n = 1$ involve only α_0, β_0, α_1, and β_1, and so on. This means that when doing calculations, we do not have to truncate the system. If we are interested in just the leading terms of the expansions (A.7), (A.8), we only need to solve Eqs. (A.10). In order to obtain the second terms, we need to add the second pair of equations, and so on.

Far away from the origin, we have

$$u(k,r) \underset{r\to\infty}{\longrightarrow} \sin(kr)A(k,\infty)+\cos(kr)B(k,\infty). \tag{A.11}$$

On the other hand, we can define the S-wave phase shift δ_0 by the asymptotics

$$u(k,r) \underset{r\to\infty}{\longrightarrow} N\sin(kr+\delta_0), \tag{A.12}$$

where N is the normalization constant. Comparing Eqs. (A.11) and (A.12), we see that $A(k,\infty)=N\cos\delta_0$, $B(k,\infty)=N\sin\delta_0$, and thus

$$\tan\delta_0(k)=\frac{B(k,\infty)}{A(k,\infty)}. \tag{A.13}$$

Therefore

$$\begin{aligned}
k\cot\delta_0 &= \frac{\alpha_0(\infty)+k^2\alpha_1(\infty)+k^4\alpha_2(\infty)+\cdots}{\beta_0(\infty)+k^2\beta_1(\infty)+k^4\beta_2(\infty)+\cdots}\\
&=\frac{\alpha_0(\infty)}{\beta_0(\infty)}+\left[\frac{\alpha_1(\infty)}{\beta_0(\infty)}-\frac{\alpha_0(\infty)\beta_1(\infty)}{\beta_0^2(\infty)}\right]k^2\\
&\quad+\frac{\alpha_0(\infty)}{\beta_0(\infty)}\left\{\frac{\alpha_2(\infty)}{\alpha_0(\infty)}-\frac{\beta_1(\infty)}{\beta_0(\infty)}\left[\frac{\alpha_1(\infty)}{\alpha_0(\infty)}-\frac{\beta_1(\infty)}{\beta_0(\infty)}\right]-\frac{\beta_2(\infty)}{\beta_0(\infty)}\right\}k^4+\cdots.
\end{aligned}$$

From this series, it is easy to find not only the first three low-energy parameters,

$$\begin{aligned}
a &= -\frac{\beta_0}{\alpha_0}\bigg|_{r=\infty},\\
r_0 &= \frac{2\alpha_0}{\beta_0}\left(\frac{\alpha_1}{\alpha_0}-\frac{\beta_1}{\beta_0}\right)\bigg|_{r=\infty},\\
V_0 &= \frac{\alpha_0}{\beta_0}\left[\frac{\alpha_2}{\alpha_0}-\frac{\beta_1}{\beta_0}\left(\frac{\alpha_1}{\alpha_0}-\frac{\beta_1}{\beta_0}\right)-\frac{\beta_2}{\beta_0}\right]\bigg|_{r=\infty},
\end{aligned}$$

of the expansion (28), but also the k-independent factors of any further terms. For this, we only need to solve an appropriate number of differential equations of the system (A.9) from $r=0$ up to a faraway point where the potential vanishes.

Appendix B. Hypercentral potential

Hypercentral potential (30) consists of the three terms

$$\langle U\rangle=\langle U_{12}\rangle+\langle U_{13}\rangle+\langle U_{23}\rangle, \tag{B.1}$$

where U_{ij} is the two-body potential acting between particles i and j. As was mentioned above, we can consider both $nn\Lambda$ and $\Lambda\Lambda n$ systems in a unified way. Let 1 and 2 be the identical particles, i.e. the nn or $\Lambda\Lambda$ pair, and 3 be the remaining Λ-particle or neutron, respectively.

The six-dimensional volume element is

$$\begin{aligned}
d\vec r_1\,d\vec r_2 &= r_1^2r_2^2\,dr_1\,dr_2\sin\theta_{\vec r_1}\,d\theta_{\vec r_1}\,d\varphi_{\vec r_1}\sin\theta_{\vec r_2}\,d\theta_{\vec r_2}\,d\varphi_{\vec r_2}\\
&= r^5\,dr\frac{1}{4}\sin^2(2\alpha)\,d\alpha\sin\theta_{\vec r_1}\,d\theta_{\vec r_1}\,d\varphi_{\vec r_1}\sin\theta_{\vec r_2}\,d\theta_{\vec r_2}\,d\varphi_{\vec r_2}\\
&= r^5\,dr\,d\omega.
\end{aligned}$$

Therefore in the five-dimensional integral (30) the volume element is

$$d\omega = \frac{1}{4}\sin^2(2\alpha)\sin\theta_{\vec r_1}\sin\theta_{\vec r_2}\,d\alpha\,d\theta_{\vec r_1}\,d\varphi_{\vec r_1}\,d\theta_{\vec r_2}\,d\varphi_{\vec r_2}. \tag{B.2}$$

Since we assume that $\ell_1 = \ell_2 = 0$ and $L = L_{\min} = 0$, the sum (7) is reduced to a single term,

$$\Phi^{jj_z}_{[L_{\min}]}(\omega) = Y_{[L_{\min}]}(\omega)\chi_{[s]}, \tag{B.3}$$

where the quantum numbers jj_z coincide with $s\sigma$. The two-body spin s_{12} of the identical pair in the S-wave state must be zero. As a result the three-body spin s is always $1/2$. The hyperspherical harmonics $Y_{[L_{\min}]}(\omega)$ is trivial (independent of the angles),

$$Y_{[L_{\min}]}(\omega) \equiv \pi^{-3/2}, \tag{B.4}$$

which means that the action of the permutation operators P^r for all three terms in Eq. (B.1) is also trivial: its eigenvalue is 1,

$$P^r_{ij}Y_{[L_{\min}]} = Y_{[L_{\min}]}, \quad ij = \{12\}, \{13\}, \{23\}. \tag{B.5}$$

The spin permutation operator P^σ_{12} for the identical pair $\{12\}$ changes the sign of $\chi_{[s]}$,

$$P^\sigma_{12}\chi_{[s]} = -\chi_{[s]}, \tag{B.6}$$

because $s_{12} = 0$ in $[s] = ((s_1s_2)s_{12}s_3)s\sigma$. For the other two pairs, its action is a bit more complicated. Indeed, recoupling the spins,

$$\begin{aligned}
\big|\big((s_1s_2)s_{12}s_3\big)s\sigma\big\rangle &= \sum_{s_{31}}\big|\big((s_3s_1)s_{31}s_2\big)s\sigma\big\rangle\big\langle\big((s_3s_1)s_{31}s_2\big)s\sigma\big|\big((s_1s_2)s_{12}s_3\big)s\sigma\big\rangle \\
&= \big|\big((s_3s_1)0s_2\big)s\sigma\big\rangle\begin{Bmatrix}\frac12 & \frac12 & 0\\ \frac12 & \frac12 & 0\end{Bmatrix} + \sqrt{3}\big|\big((s_3s_1)1s_2\big)s\sigma\big\rangle\begin{Bmatrix}\frac12 & \frac12 & 0\\ \frac12 & \frac12 & 1\end{Bmatrix} \\
&= -\frac{1}{2}\big|\big((s_3s_1)0s_2\big)s\sigma\big\rangle + \frac{\sqrt{3}}{2}\big|\big((s_3s_1)1s_2\big)s\sigma\big\rangle,
\end{aligned}$$

we find that

$$\chi^+_{[s]}P^\sigma_{13}\chi_{[s]} = \frac{1}{2}. \tag{B.7}$$

Similarly, it is easy to find for the pair $\{23\}$ that

$$\chi^+_{[s]}P^\sigma_{23}\chi_{[s]} = \frac{1}{2}. \tag{B.8}$$

When inserting the potentials U_{ij} given by Eq. (22), into the integral (30), we should use the following interparticle distances (see Fig. 1),

$$\rho_{12} = r\sqrt{\frac{M}{\mu_1}}\cos\alpha, \tag{B.9}$$

$$\rho_{13} = r\sqrt{\frac{M}{\mu_2}\sin^2\alpha + \frac{M}{4\mu_1}\cos^2\alpha - \frac{M}{2\sqrt{\mu_1\mu_2}}\sin(2\alpha)\cos\theta_{\vec r_2}}, \tag{B.10}$$

$$\rho_{23} = r\sqrt{\frac{M}{\mu_2}\sin^2\alpha + \frac{M}{4\mu_1}\cos^2\alpha + \frac{M}{2\sqrt{\mu_1\mu_2}}\sin(2\alpha)\cos\theta_{\vec r_2}}. \tag{B.11}$$

Since the particles 1 and 2 are identical the interactions U_{13} and U_{23} are the same. Moreover, according to Eqs. (B.7), (B.8), the product $\chi^+_{[s]} U_{13} \chi_{[s]}$ has the same dependence on ρ_{13} as $\chi^+_{[s]} U_{23} \chi_{[s]}$ depends on ρ_{23}. Actually, they depend on ρ^2_{13} and ρ^2_{23} because all the terms in the potential (22) are of the Gaussian form. Comparing Eqs. (B.10) and (B.11), we see that the integrands for $\langle U_{13}\rangle$ and $\langle U_{23}\rangle$ differ only in the sign of the power of the exponential factors corresponding to the last terms of ρ^2_{13} and ρ^2_{23}. This difference however has no effect on the integrals. Indeed, the integration over $\theta_{\vec r_2}$,

$$\int_0^\pi \exp(\pm f \cos\theta_{\vec r_2}) \sin\theta_{\vec r_2}\, d\theta_{\vec r_2} = \int_{-1}^{1} \exp(\pm f t)\, dt = \frac{1}{f}\big(e^f - e^{-f}\big) = \frac{2}{f}\sinh(f),$$

gives the same result for both signs. Therefore $\langle U_{13}\rangle = \langle U_{23}\rangle$ and hence

$$\langle U\rangle = \langle U_{12}\rangle + 2\langle U_{13}\rangle. \tag{B.12}$$

Performing trivial integrations over $\varphi_{\vec r_1}$, $\varphi_{\vec r_2}$, $\theta_{\vec r_1}$, and $\theta_{\vec r_2}$ (trivial in the case of $\langle U_{12}\rangle$), we obtain the following expressions for the terms of the hypercentral potential (B.12),

$$\langle U_{12}\rangle = \frac{4}{\pi}\int_0^{\pi/2} d\alpha \sin^2(2\alpha)\big[W_1^{\{12\}} \exp\big(-a_1^{\{12\}}\eta r^2\big) - W_3^{\{12\}} \exp\big(-a_3^{\{12\}}\eta r^2\big)\big], \tag{B.13}$$

$$\begin{aligned}\langle U_{13}\rangle = \frac{2}{\pi}\int_0^{\pi/2} d\alpha \sin^2(2\alpha)\big[&W_1^{\{13\}} \exp\big(-a_1^{\{13\}}\zeta r^2\big) s\big(a_1^{\{13\}}\xi r^2\big)\\ &- \frac{3}{4} W_2^{\{13\}} \exp\big(-a_2^{\{13\}}\zeta r^2\big) s\big(a_2^{\{13\}}\xi r^2\big)\\ &- \frac{1}{4} W_3^{\{13\}} \exp\big(-a_3^{\{13\}}\zeta r^2\big) s\big(a_3^{\{13\}}\xi r^2\big)\big],\end{aligned} \tag{B.14}$$

where

$$\eta(\alpha) = \frac{M}{\mu_1}\cos^2\alpha, \qquad \zeta(\alpha) = \frac{M}{\mu_2}\sin^2\alpha + \frac{M}{4\mu_1}\cos^2\alpha,$$
$$\xi(\alpha) = \frac{M}{2\sqrt{\mu_1\mu_2}}\sin(2\alpha), \qquad s(f) = \frac{1}{f}\big(e^f - e^{-f}\big).$$

The parameters $W_n^{\{ij\}}$ and $a_n^{\{ij\}}$, where the symbol $\{ij\}$ means a choice of the pair of interacting particles, are given in Table 1. For each (complex) value of the hyperradius r, which was needed in our calculations, the integrals (B.13) and (B.14) were evaluated numerically.

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