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Coherent photoproduction of η -mesons on three-nucleon systems

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Abstract

A microscopic few-body description of near-threshold coherent photoproduction of the η meson on tritium and ^3He targets is given. The photoproduction cross section is calculated using the Finite Rank Approximation (FRA) of the nuclear Hamiltonian. The results indicate a strong final state interaction of the η meson with the residual nucleus. Sensitivity of the results to the choice of the ηN T -matrix is investigated.

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1. Introduction

Investigations of the η -nucleus interaction are motivated by various reasons. Some of them, such as the possibility of forming quasi-bound states or resonances [1] in the η -nucleus system, are purely of nuclear nature. The others are related to the study of the properties and structure of the $S_{11}(1535)$ resonance which is strongly coupled to the ηN channel.

For example, it is interesting to investigate the behavior of the η -meson in nuclear media where, after colliding with the nucleons, it readily forms the S_{11} resonance. The interaction of this resonance with the surrounding nucleons can be described in different ways [2],

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depending on whether the structure of this resonance is defined in terms of some quark configurations or by the coupling of meson–baryon channels, as suggested in Ref. [3,4]. The estimation by Tiwari et al. [5] shows, that in case of pseudoscalar ηNN coupling there is an essential density dependent reduction of the η -meson mass and of the η – η' mixing angle.

The importance of the influence of the nuclear medium on the mesons passing through it, was recently emphasized by Drechsel et al. [6]. If this influence is described in terms of self-energies and effective masses, then the passing of π -mesons through the nucleus provides “saturation” of the isobar propagator (or self-energy). This phenomenon manifests itself even in light nuclei [6]. Similar ideas were discussed also in Ref. [7]. In other words, the propagation of η -mesons inside the nucleus is a new challenge for theorists.

Another interesting issue related to the η -nucleus interaction is the study of charge symmetry breaking, which may partly be attributed to the η – π^0 mixing (see, for example, Refs. [8–11]). In principle, one can extract the value of the mixing angle from experiments involving η -nucleus interaction and compare the results with the predictions of quark models. However, to do such an extraction, one has to make an extrapolation of the η -nucleus scattering amplitude into the area of unphysical energies below the η -nucleus threshold. This is a highly model dependent procedure requiring a reliable treatment of the η -nucleus dynamics.

In this respect, few-body systems such as ηd , $\eta^3\text{He}$, and $\eta^4\text{He}$, have obvious advantages since they can be treated using rigorous Faddeev-type equations. To the best of our knowledge, the exact AGS theory [16] has been used in the few calculations (see Refs. [12–15]) for the ηd and in one recent calculation [17] for the $\eta^3\text{H}$ and $\eta^3\text{He}$ systems.

A solution of the few-body equations presupposes the knowledge of the corresponding two-body T -matrices $t_{\eta N}$ and t_{NN} off the energy shell. Due to the fact that at low energies the η meson interacts with a nucleon mainly via the formation of the S_{11} -resonance, the inclusion of the higher partial waves ($\ell > 0$) is unnecessary. Furthermore, since the ηN interaction is poorly known, the effect of the fine tuned details of the “realistic” NN potentials would be far beyond the level of the overall accuracy of the ηA theory.

In contrast to the well-established NN forces, the ηN interaction is constructed using very limited information available, namely, the ηN scattering length and the parameters of the S_{11} -resonance. Furthermore, only the resonance parameters are known more or less accurately while the scattering length (which is complex) is determined with large uncertainties. Moreover, practically nothing is known about the off-shell behavior of the ηN amplitude. It is simply assumed that the off-shell behavior of this amplitude could be approximated (like in the case of π mesons) by appropriate Yamaguchi form-factors (see, for example, Refs. [12–15,18,19]). However, if the available data are used to construct a potential via, for example, Fiedeldey’s inverse scattering procedure [20], the resulting form factor of the separable potential is not that simple. The problem becomes even more complicated due to the multichannel character of the ηN interaction with the additional off-shell uncertainties stemming from the π -meson channel.

In such a situation, it is desirable to narrow as much as possible the uncertainty intervals for the parameters of ηN interaction. This could be done by demanding consistency of theoretical predictions based on these parameters, with existing experimental data for two-, three-, and four-body η -nucleus processes. This is one of the objectives of the present work.

To do this, we calculate the cross sections of coherent η -photoproduction on ${}^3\text{He}$ and ${}^3\text{H}$ and study their sensitivity to the parameters of the ηN amplitude.

2. Formalism

We start by assuming that the Compton scattering on a nucleon,

$$\gamma + N \rightarrow N + \gamma,$$

as well as the processes of multiple re-appearing of the photon in the intermediate states,

$$\gamma + N \rightarrow N + \eta \rightarrow \gamma + N \rightarrow N + \eta \rightarrow \dots,$$

give a negligible contribution to the coherent η -photoproduction on a nucleus A . In our model, we also neglect virtual excitations and breakup of the nucleus immediately after its interaction with the photon. With these assumptions, the process

$$\gamma + A \rightarrow A + \eta \tag{1}$$

can be formally described in two steps: in the first one, the photon produces the η meson on one of the nucleons,

$$\gamma + N \rightarrow N + \eta, \tag{2}$$

in the second step (final state interaction) the η meson is elastically scattered off the nucleus,

$$\eta + A \rightarrow A + \eta. \tag{3}$$

An adequate treatment of the scattering step is, of course, the most difficult and crucial part of the theory. The first microscopic calculations concerning the low-energy scattering of the η -meson from ${}^3\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$ were done in Refs. [21–27] where the few-body dynamics of these systems was treated by employing the Finite-Rank Approximation (FRA) [28] of the nuclear Hamiltonian. This approximation consists in neglecting the continuous spectrum in the spectral expansion

$$H_A = \sum_n \mathcal{E}_n |\psi_n\rangle \langle \psi_n| + \text{continuum}$$

of the Hamiltonian H_A describing the nucleus. Since the three- and four-body nuclei have only one bound state, FRA reduces to

$$H_A \approx \mathcal{E}_0 |\psi_0\rangle \langle \psi_0|. \tag{4}$$

Physically, this means that we exclude the virtual excitations of the nucleus during its interaction with the η meson. It is clear that the stronger the nucleus is bound, the smaller is the contribution from such processes to the elastic ηA scattering. By comparing with the results of the exact AGS calculations, it was shown [29] that even for ηd scattering, having the weakest nuclear binding, the FRA method works reasonably well, which implies that we obtain sufficiently accurate results by applying this method to the η ${}^3\text{H}$, η ${}^3\text{He}$, and even more so to the η ${}^4\text{He}$ scattering.

In essence, the FRA method can be described as follows (for details see Ref. [28]). Let

$$H = h_0 + V + H_A$$

be the total ηA Hamiltonian, where h_0 describes the free η -nucleus motion and

$$V = \sum_{i=1}^A V_i$$

the sum of the two-body η -nucleon potentials. The Lippmann–Schwinger equation

$$T(z) = \sum_{i=1}^A V_i + \sum_{i=1}^A V_i(z - h_0 - H_A)^{-1} T(z) \quad (5)$$

for the η -nucleus T -matrix can be rewritten as

$$T(z) = W(z) + W(z)M(z)T(z), \quad (6)$$

where

$$M(z) = G_0(z)H_A G_A(z), \quad (7)$$

$$G_0(z) = (z - h_0)^{-1}, \quad (8)$$

$$G_A(z) = (z - h_0 - H_A)^{-1}, \quad (9)$$

and the auxiliary operator $W(z)$ is split into A components of Faddeev-type

$$W(z) = \sum_{i=1}^A W_i(z), \quad (10)$$

satisfying the following system of equations

$$W_i(z) = t_i(z) + t_i(z)G_0(z) \sum_{j \neq i}^A W_j(z) \quad (11)$$

with t_i being the two-body T -matrix describing the interaction of the η -meson with the i th nucleon, i.e.,

$$t_i(z) = V_i + V_i G_0(z) t_i(z). \quad (12)$$

It should be emphasized that up to this point no approximation has been made and, therefore, the set of equations (6)–(12) is equivalent to the initial equation (5). However, to solve Eq. (6), we have to resort to the approximation (4) which simplifies its kernel (7) to

$$M(z) \approx \frac{\mathcal{E}_0 |\psi_0\rangle \langle \psi_0|}{(z - h_0)(z - \mathcal{E}_0 - h_0)}. \quad (13)$$

With this approximation, the sandwiching of Eq. (6) between $\langle \psi_0|$ and $|\psi_0\rangle$ and the partial wave decomposition give a one-dimensional integral equation for the amplitude of the process (3). Although this one-dimensional equation may look similar to the integral equation of the first-order optical-potential theory, the FRA approach is quite different. Firstly, in contrast to the optical potential of the first order, the operator $W(z)$ includes all

orders of rescattering via solution of Eq. (11). Secondly, the ηN amplitudes $t_i(z)$ entering Eq. (11), are taken as operators in the many-body space and off the energy shell (note that G_0 in Eq. (12) is four-body propagator with ηA reduced mass and z is total four-body energy), i.e., the FRA method does not involve the so-called “impulse approximation” (using free two-body amplitudes for t_i) which is an indispensable part of the optical theory.

The question then arises how a photon can be included in this formalism in order to describe the photoproduction process (1). This can be achieved by following the same procedure as in Ref. [30] where the reaction (1) with $A = 2$ was treated within the framework of the exact AGS equations, and the photon was introduced by considering the ηN and γN states as two different channels of the same system. This implies that the operators t_i should be replaced by 2×2 matrices. It is clear that such replacements of the kernels of the integral equations (11) and subsequently of the integral equation (6) lead to solutions having a similar matrix form

$$t_i \rightarrow \begin{pmatrix} t_i^{\gamma\gamma} & t_i^{\gamma\eta} \\ t_i^{\eta\gamma} & t_i^{\eta\eta} \end{pmatrix} \implies W_i \rightarrow \begin{pmatrix} W_i^{\gamma\gamma} & W_i^{\gamma\eta} \\ W_i^{\eta\gamma} & W_i^{\eta\eta} \end{pmatrix} \implies T \rightarrow \begin{pmatrix} T^{\gamma\gamma} & T^{\gamma\eta} \\ T^{\eta\gamma} & T^{\eta\eta} \end{pmatrix}. \tag{14}$$

Here $t_i^{\gamma\gamma}$ describes the Compton scattering, $t_i^{\eta\gamma}$ the photoproduction process, and $t_i^{\eta\eta}$ the elastic η scattering on the i th nucleon. What is finally needed is the cross section

$$\frac{d\sigma}{d\Omega} = \frac{\mu_{\eta A}}{(2\pi)^2} \frac{k_\eta}{k_\gamma} \frac{E_\gamma m_A}{E_\gamma + m_A} \frac{1}{4} \sum_{s'_z, s_z, \epsilon} \left| \langle \vec{k}_\eta, \varphi \phi_{s'_z, t_z}^a | T^{\eta\gamma} (\mathcal{E}_0 + E_\gamma) | \varphi \phi_{s_z, t_z}^a, \vec{k}_\gamma, \epsilon \rangle \right|^2 \tag{15}$$

of the reaction (1) averaged over orientations s_z of the initial nuclear spin and photon polarization ϵ and summed over spin orientations s'_z in the final state. Here φ and ϕ_{s_z, t_z}^a are the spatial and spin–isospin parts of ψ_0 (with the third components of the nuclear spin and isospin being s_z and t_z respectively), \vec{k}_γ and \vec{k}_η are the momenta of the photon and η meson, E_γ is the energy of the photon, m_A the mass of the nucleus, and $\mu_{\eta A}$ the reduced mass of the meson and the nucleus.

However, it is technically more convenient to consider radiative η -absorption, i.e., the inverse reaction. Then the photoproduction cross section can be obtained by applying the principle of detailed balance. The reason for this is that all the processes in which the photon appears more than once, i.e., the terms of the integral equations of type $W^{\gamma\gamma} M T^{\gamma\eta}$ or $W^{\eta\gamma} M T^{\gamma\eta}$ involving more than one electromagnetic vertex, can be neglected. Omission of these terms in (6) results in decoupling the elastic scattering equation

$$T^{\eta\eta} = W^{\eta\eta} + W^{\eta\eta} M T^{\eta\eta} \tag{16}$$

from the equation for the radiative absorption

$$T^{\gamma\eta} = W^{\gamma\eta} + W^{\gamma\eta} M T^{\eta\eta}. \tag{17}$$

Once $T^{\eta\eta}$ is calculated, the radiative absorption T -matrix (17) can be obtained by integration.

Therefore, the procedure of calculating the photoproduction cross section (15) consists of the following steps.

- Solving the system of equations

$$W_i^{\eta\eta} = t_i^{\eta\eta} + t_i^{\eta\eta} G_0 \sum_{j \neq i}^A W_j^{\eta\eta} \quad (18)$$

for the auxiliary elastic-scattering operators $W_i^{\eta\eta}$.

- Calculating (by integration) the auxiliary matrices $W_i^{\gamma\eta}$ from

$$W_i^{\gamma\eta} = t_i^{\gamma\eta} + t_i^{\gamma\eta} G_0 \sum_{j \neq i}^A W_j^{\eta\eta}. \quad (19)$$

- Solving the integral equation

$$T^{\eta\eta} = \sum_{i=1}^A W_i^{\eta\eta} + \sum_{i=1}^A W_i^{\eta\eta} M T^{\eta\eta} \quad (20)$$

for the elastic-scattering T -matrix.

- Calculating (by integration) the radiative absorption T -matrix

$$T^{\gamma\eta} = \sum_{i=1}^A W_i^{\gamma\eta} + \sum_{i=1}^A W_i^{\gamma\eta} M T^{\eta\eta}. \quad (21)$$

- Substituting this T -matrix into Eq. (15) to obtain the differential cross section for the photoproduction. This is possible because the absolute values of the photoproduction and radiative absorption T -matrices coincide.

3. Two-body interactions

To implement the steps described in the previous section, we need the two-body T -matrices $t^{\eta\eta}$ and $t^{\gamma\eta}$ for the elastic ηN scattering and the radiative absorption $N(\eta, \gamma)N$ on a single nucleon, respectively. Furthermore, all equations (18)–(21) have to be sandwiched between $\langle \psi_0 |$ and $|\psi_0\rangle$ (ground state of the nucleus). Since at low energies both the elastic scattering and photoproduction of the η meson on a nucleon proceed mainly via formation of the S_{11} resonance, we may retain only the S -waves in the partial wave expansions of the corresponding two-body T -matrices.

3.1. Elastic ηN scattering

The problem of constructing an ηN potential or directly the corresponding T -matrix $t^{\eta\eta}$ has no unique solution since the only experimental information available consists of the S_{11} -resonance pole position $E_0 - i\Gamma/2$ and the ηN scattering length $a_{\eta N}$. In the present work we use three different versions of $t^{\eta\eta}$.

Version I

Without any scattering data it is practically impossible to construct a reliable ηN potential. In the low-energy region, however, the elastic scattering can be viewed as the process of formation and subsequent decay of S_{11} resonance, i.e.,

$$\eta + N \rightarrow S_{11} \rightarrow N + \eta. \tag{22}$$

This implies that the corresponding Breit–Wigner formula could be a good approximation for the ηN cross section. Therefore, we may adopt the following ansatz

$$t^{\eta\eta}(k', k; z) = g(k')\tau(z)g(k) \tag{23}$$

where the propagator $\tau(z)$, describing the intermediate state of the process (22), is assumed to have a simple Breit–Wigner form

$$\tau(z) = \frac{\lambda}{z - E_0 + i\Gamma/2}, \tag{24}$$

which guaranties that the T -matrix (23) has a pole at the proper place. The vertex function $g(k)$ for the processes $\eta N \leftrightarrow S_{11}$ is chosen to be

$$g(k) = (k^2 + \alpha^2)^{-1} \tag{25}$$

which in configuration space is of Yukawa-type. The range parameter $\alpha = 3.316 \text{ fm}^{-1}$ was determined in Ref. [31] while the parameters of the S_{11} -resonance

$$E_0 = 1535 \text{ MeV} - (m_N + m_\eta), \quad \Gamma = 150 \text{ MeV}$$

are taken from Ref. [32]. The strength parameter λ is chosen to reproduce the η -nucleon scattering length $a_{\eta N}$,

$$\lambda = 2\pi \frac{\alpha^4 (E_0 - i\Gamma/2)}{\mu_{\eta N}} a_{\eta N}, \tag{26}$$

the imaginary part of which accounts for the flux losses into the πN channel. Here $\mu_{\eta N}$ is the ηN reduced mass.

The two-body scattering length $a_{\eta N}$ is not accurately known. Different analyses [33] provided values for $a_{\eta N}$ in the range

$$0.27 \leq \text{Re } a_{\eta N} \leq 0.98 \text{ fm}, \quad 0.19 \leq \text{Im } a_{\eta N} \leq 0.37 \text{ fm}. \tag{27}$$

In most recent publications the value used for $\text{Im } a_{\eta N}$ is around 0.3 fm. However, for $\text{Re } a_{\eta N}$ the estimates are still very different (compare, for example, Refs. [34] and [35]). In the present work we assume that

$$a_{\eta N} = (0.75 + i0.27) \text{ fm}. \tag{28}$$

The T -matrix $t^{\eta\eta}$ constructed in this way reproduces the scattering length (28) and the S_{11} pole, but apparently violates the two-body unitarity since it does not obey the two-body Lippmann–Schwinger equation.

Version II

An alternative way of constructing the two-body T -matrix $t^{\eta\eta}$ is to solve the corresponding Lippmann–Schwinger equation with an appropriate separable potential having the same form-factors (25). However, a one-term separable T -matrix obtained in this way, does not have a pole at $z = E_0 - i\Gamma/2$. To recover the resonance behavior in this case, we use the trick suggested in Ref. [19], namely, we use an energy-dependent strength of the potential

$$V(k, k'; z) = g(k) \left[\Lambda + C \frac{\zeta}{\zeta - z} \right] g(k'),$$

where Λ is complex while C and ζ are real constants. With this ansatz for the potential, the Lippmann–Schwinger equation gives the T -matrix in the form (23) with

$$\tau(z) = - \left(\frac{4\pi\alpha^3}{\mu_{\eta N}} \right) \frac{\Lambda(\zeta - z) + C\zeta}{\zeta - z - [\Lambda(\zeta - z) + C\zeta] / (1 - i\sqrt{2z\mu_{\eta N}}/\alpha)^2}. \quad (29)$$

The constants Λ , C , and ζ can be chosen in such a way that the corresponding scattering amplitude reproduces the scattering length $a_{\eta N}$ and has a pole at $z = E_0 - i\Gamma/2$. This version of $t^{\eta\eta}$ also reproduces the scattering length (28) and the S_{11} pole. Moreover, it is consistent with the condition of two-body unitarity.

Version III

We can also construct $t^{\eta\eta}$ in the form (23), with the same $\tau(z)$ as in (24), but obeying the unitarity condition

$$(1 - 2\pi i t^{\eta\eta})(1 - 2\pi i t^{\eta\eta})^\dagger = 1. \quad (30)$$

Of course, with the simple form (23) we cannot satisfy the condition (30) at all energies. To simplify the derivations, we impose this condition on $t^{\eta\eta}$ at $z = E_0$. Since Eq. (30) is real, it can fix only one parameter and we need one more condition to fix both the real and imaginary parts of the complex λ . As the second equation, we used the real or imaginary part of Eq. (26) (version III(a) or III(b), respectively) with $a_{\eta N}$ given by (28).

This procedure guaranties two-body unitarity and gives the correct position of the resonance pole, but the resulting $t^{\eta\eta}$ provides a value of $a_{\eta N}$ which, of course, slightly differs from (28), namely,

$$a_{\eta N} = (0.77 + i0.26) \text{ fm}, \quad \text{version III(a)}, \quad (31)$$

$$a_{\eta N} = (0.79 + i0.32) \text{ fm}, \quad \text{version III(b)}. \quad (32)$$

In what follows we use these three versions of the matrix $t^{\eta\eta}$. All of them have the same separable form (23) but different $\tau(z)$. Comparison of the results obtained with these three T -matrices can give an indication of the importance of two-body unitarity in the photoproduction processes.

3.2. Radiative absorption $N(\eta, \gamma)N$

In constructing the radiative absorption T -matrix $t^{\gamma\eta}$, the S_{11} dominance in the near-threshold region also plays an important role. It was experimentally shown [36] that, at low energies, the reaction (2) proceeds mainly via formation of the S_{11} -resonance,

$$\gamma + N \rightarrow S_{11} \rightarrow N + \eta, \tag{33}$$

by the S -wave E_{0+} multipole. This means that in the standard CGLN decomposition of the $N(\gamma, \eta)N$ amplitude (see, for example, Ref. [31]) only the term proportional to the dot-product ($\vec{\sigma} \cdot \vec{\epsilon}$) of the nucleon spin and photon polarization can be retained, i.e.,

$$t^{\gamma\eta} = f^{\gamma\eta}(\vec{\sigma} \cdot \vec{\epsilon}). \tag{34}$$

The dominance of the process (33) implies that $f^{\gamma\eta}$ in this energy region can be written in a separable form similar to (23). To construct such a separable T -matrix, we use the results of Ref. [37] where $t^{\gamma\eta}$ was considered as an element of a multi-channel T -matrix which simultaneously describes experimental data for the processes

$$\begin{aligned} \pi + N &\rightarrow \pi + N, & \pi + N &\rightarrow \eta + N, \\ \gamma + N &\rightarrow \pi + N, & \gamma + N &\rightarrow \eta + N \end{aligned}$$

on the energy shell in the S_{11} -channel (the ηN scattering length obtained in Ref. [37] is the same as we use for constructing versions I and II of $t^{\eta\eta}$). In the present work, we take the T -matrix $t_{\text{on}}^{\gamma\eta}(E)$ from Ref. [37] and extend it off the energy shell via

$$f_{\text{off}}^{\gamma\eta}(k', k; E) = \frac{\kappa^2 + E^2}{\kappa^2 + k'^2} t_{\text{on}}^{\gamma\eta}(E) \frac{\alpha^2 + 2\mu_{\eta N} E}{\alpha^2 + k^2}, \tag{35}$$

where κ is a parameter. The Yamaguchi form-factors used in this ansatz go to unity on the energy shell. Since κ is not known, this parameter is varied in our calculations within an interval $1 < \kappa < 10 \text{ fm}^{-1}$ which is a typical range for meson–nucleon forces. It is known that $t^{\gamma\eta}$ is different for neutron and proton. In this work we assume that they have the same functional form (35) but differ by a constant factor,

$$t_n^{\gamma\eta} = A t_p^{\gamma\eta}.$$

Multipole analysis [38] gives for this factor the estimate $A = -0.84 \pm 0.15$. Therefore, if we direct the z -axis along the photon momentum \vec{k}_γ , the radiative absorption T -matrix entering our equations, can be written as

$$t^{\gamma\eta} = f_{\text{off}}^{\gamma\eta} \cdot (\sigma_x \epsilon_x + \sigma_y \epsilon_y) (P_p + A P_n), \tag{36}$$

where ϵ_x and ϵ_y are transverse components of the photon polarization vector while P_p and P_n are the operators projecting onto the proton and neutron isotopic states, respectively.

3.3. Nuclear subsystem

Since the T -matrices $t^{\eta\eta}$ and $t^{\gamma\eta}$ are poorly known and their uncertainties significantly limit the overall accuracy of the theory, it is not necessary to use any sophisticated

(“realistic”) potential to describe the NN interaction. Therefore we may safely assume that the nucleons interact with each other only in the S -wave state.

To obtain the necessary nuclear wave function ψ_0 , we solve the few-body equations of the Integro-Differential Equation Approach (IDEA) [39,40] with the Malfliet–Tjon potential [41]. This approach is based on the Hyperspherical Harmonic expansion method applied to Faddeev-type equations. In fact, in the case of S -wave potentials, the IDEA is fully equivalent to the exact Faddeev equations. Therefore, the bound states used in our calculations are derived, to all practical purposes, via an exact formalism.

4. Spin–isospin average

The wave function $\psi_0 = \varphi\phi_{s_z, t_z}^a$ of the ${}^3\text{H}/{}^3\text{He}$ system obtained by solving the IDEA equations with the Malfliet–Tjon potential, has only the symmetric S -wave spatial component φ multiplied by the antisymmetric spin–isospin part

$$\phi_{s_z, t_z}^a = \frac{1}{\sqrt{2}}(\chi'_{s_z} \eta''_{t_z} - \chi''_{s_z} \eta'_{t_z}),$$

where χ' , χ'' and η' , η'' are the mixed symmetry states in the spin and isospin sub-spaces. The matrix element of $T^{\gamma\eta}$ in Eq. (15) involves the average not only over the spatial part of ψ_0 but over ϕ^a as well. The average $\langle \phi^a | T^{\gamma\eta} | \phi^a \rangle$ can be done before we start solving Eqs. (18)–(21).

Since $t^{\eta\eta}$, G_0 , and M do not involve spin–isospin operators, the averaging of Eqs. (18) and (20) over ϕ^a is trivial: it does not produce any additional coefficients. Eq. (19), however, changes. Indeed, for each nucleon ($i = 1, 2, 3$), it involves the operator (36) which causes nucleon spin to flip over.

Formal averaging of $t_j^{\gamma\eta}$ (for $j = 1, 2, 3$) over the states ϕ_{s_z, t_z}^a having definite values of the z -components of total spin (s_z) and isospin (t_z), gives the same results

$$\langle \phi_{s'_z, t'_z}^a | t_j^{\gamma\eta} | \phi_{s_z, t_z}^a \rangle = \begin{cases} \delta_{-s'_z, s_z} f_{\text{off}}^{\gamma\eta} \frac{A}{3} (\epsilon_x + i\epsilon_y), & \text{for } t_z = +1/2 \quad ({}^3\text{He}), \\ \delta_{-s'_z, s_z} f_{\text{off}}^{\gamma\eta} \frac{1}{3} (\epsilon_x + i\epsilon_y), & \text{for } t_z = -1/2 \quad ({}^3\text{H}) \end{cases} \quad (37)$$

for all three nucleons. This means that all three matrix elements $\langle \phi_{s'_z, t'_z}^a | W_j^{\gamma\eta} | \phi_{s_z, t_z}^a \rangle$ ($j = 1, 2, 3$) acquire the same coefficient, namely, $A(\epsilon_x + i\epsilon_y)/3$ or $(\epsilon_x + i\epsilon_y)/3$ depending on t_z . Via Eq. (21), the same coefficient goes to the matrix element $\langle \phi_{s'_z, t'_z}^a | T^{\gamma\eta} | \phi_{s_z, t_z}^a \rangle$. Since $\epsilon_x^2 + \epsilon_y^2 = 1$, this gives the factor $|A|^2/9$ (for the case of ${}^3\text{He}$) or $1/9$ (for the case of ${}^3\text{H}$) in the final Eq. (15) for the photoproduction cross section. Thus, the cross section for the ${}^3\text{He}$ target quadratically depends on A while for the case of ${}^3\text{H}$ it is independent of A .

5. Results and discussion

Figs. 1 and 2 show the results of our calculations for the total cross section of the coherent process (1). The calculations were done for two nuclear targets, ${}^3\text{H}$ and ${}^3\text{He}$,

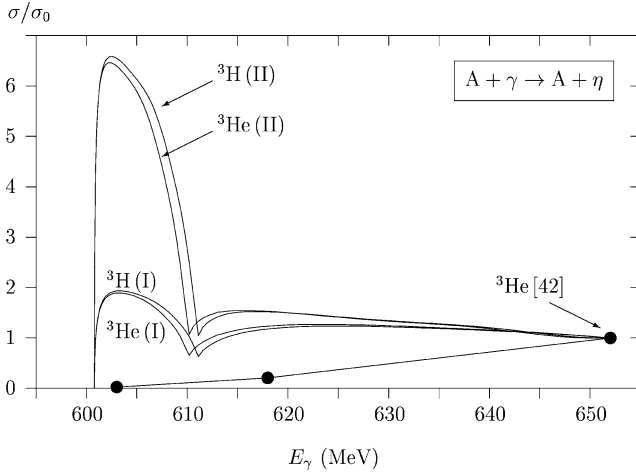


Fig. 1. Normalized cross section of the coherent η -photoproduction on the ${}^3\text{H}$ and ${}^3\text{He}$ targets, calculated with the two versions of $t^{\eta\eta}$ which are denoted as (I) and (II), respectively. All curves correspond to $a_{\eta N} = (0.75 + i0.27)$ fm, $\kappa = \alpha = 3.316$ fm $^{-1}$, and $A = -0.84$. The circles represent the points calculated in Ref. [42] for the ${}^3\text{He}$ target within the optical model. Each curve is normalized to its own σ_0 , the value of σ at $E_\gamma = 652$ MeV.

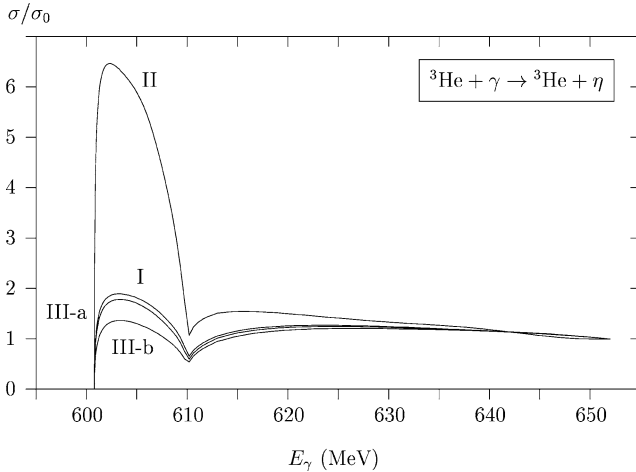


Fig. 2. Normalized cross section of the coherent η -photoproduction on ${}^3\text{He}$, calculated with the four versions of $t^{\eta\eta}$ which are denoted as (I), (II), (III(a)), and (III(b)), respectively. All four curves correspond to $\kappa = \alpha = 3.316$ fm $^{-1}$ and $A = -0.84$. For the curves (I) and (II) the $a_{\eta N}$ is given by Eq. (28) while for the (III(a)) and (III(b)) curves by Eqs. (31) and (32).

using the three versions of $t^{\eta\eta}$ described in the Section 3.1. The curves corresponding to these three T -matrices are denoted by (I), (II), and (III(a), III(b)), respectively.

We found that the coherent η -photoproduction on these targets is strongly enhanced in the near-threshold region as compared to higher photon energies ($E_\gamma > 610$ MeV).

Table 1

Values of the total cross section of the coherent process (I) at $E_\gamma = 652$ MeV calculated with the four versions of $t^{\eta\eta}$ which are denoted as (I), (II), (III(a)), and (III(b)). These values are used to normalize the curves shown in Figs. 1 and 2

Target	σ_0 (I), nb	σ_0 (II), nb	σ_0 (III(a)), nb	σ_0 (III(b)), nb	σ_0 (Ref. [42]), nb
^3H	49.33	30.58			
^3He	34.54	21.70	33.48	30.48	59.81

This can be attributed to strong final state interaction caused, for example, by a pole of the scattering S -matrix, situated in the complex-energy plane not far from the threshold energy, or in other words, to formation of η -nucleus resonance. In order to emphasize this finding and to remove the insignificant but distracting differences among different curves, we present the results in a normalized form. Each curve shows the ratio $\sigma(E_\gamma)/\sigma_0$ with σ_0 being the corresponding cross section at $E_\gamma = 652$ MeV, i.e., at the energy where the near-threshold enhancement dies out. At this energy, all the curves become flat and are not far from each other as well as from the value of 59.812 nb obtained in Ref. [42]. The normalization values σ_0 are given in Table 1.

As can be seen in Fig. 1, the two versions of $t^{\eta\eta}$, (I) and (II), give significantly different results despite the fact that both of them reproduce the same $a_{\eta N}$ and the S_{11} -resonance. This indicates that the scattering of the η meson on the nucleons (final state interaction) is very important in the description of the near-threshold photoproduction process. This conclusion is further substantiated when comparing our curves with the corresponding points (circles) calculated for the ^3He target in Ref. [42]. There the final state interaction was treated using an optical potential of the first order. It is well known that the first-order optical theory is not adequate at energies near resonances. This is the reason why the calculations of Ref. [42] underestimate σ near the threshold where, with $a_{\eta N} = (0.75 + i0.27)$ fm, the systems $\eta^3\text{H}$ and $\eta^3\text{He}$ show a resonance behavior [24].

Significant differences between the corresponding curves (I) and (II) in Fig. 1 imply that two-body unitarity is important as well. Actually, due to the resonant character of the final state interaction, all the details of $t^{\eta\eta}$ have strong influence on the photoproduction cross section in the near-threshold region. Fig. 2 where we compare the results corresponding to the three choices of $\tau(z)$ in (23), serve as another illustration of this statement.

Since nothing is known about the parameter κ , we assume $\kappa = \alpha$ as its basic value. This can be motivated by the fact that both the elastic scattering and radiative absorption (photoproduction) of the η meson on the nucleon go via formation of the same S_{11} resonance. This means that at least one vertex, namely, $\eta N \leftrightarrow S_{11}$ should be the same for both the elastic scattering and radiative absorption.

To find out how crucial the choice of κ is, we did two additional calculations with $\kappa = 1 \text{ fm}^{-1}$ and $\kappa = 10 \text{ fm}^{-1}$. We found that even with this wide variation, the corresponding $\sigma(E_\gamma)$ curves show practically identical enhancement of the cross section (less than 1% difference). The cross section only slightly increases when the range of the interaction becomes smaller (when κ grows). Therefore, the dependence on κ is not very strong and the choice $\kappa = \alpha$ gives a reasonable estimate for the photoproduction cross section.

As far as the dependence of σ on the choice of the parameter $A = t_n^{\eta\eta}/t_p^{\eta\eta}$ is concerned, we found (see Section 4) that for η photoproduction on ^3H the cross section in our model

does not depend on A , while for the ${}^3\text{He}$ target the A -dependence is quadratic. This means that among these two nuclei, the helium is preferable candidate for experimental determination of the ratio A . The sign or any phase factor of A , however, has no influence on the cross section if the electromagnetic vertex is taken into account only in the first order as it was done in our calculation.

The cusp exhibited by all the curves at the threshold of total nuclear break-up reflects losses of the flux into the non-coherent channel.

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