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Low-energy ηd-resonance

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Abstract. Elastic ηd-scattering is considered within the Alt-Grassberger-Sandhas (AGS) formalism for various ηN input data. A three-body resonant state is found close to the ηd threshold. This resonance is sustained for different choices of the two-body ηN-scattering length $a_{\eta N}$. The position of the resonance moves towards the ηd threshold when $\mathrm{Re}\, a_{\eta N}$ is increased, and turns into a quasi-bound state at $\mathrm{Re}\, a_{\eta N} \sim 0.7$–$0.8$ fm depending on the choice of $\mathrm{Im}\, a_{\eta N}$.

PACS. 25.80.-e Meson and hyperon-induced reaction – 21.45.+v Few-body systems – 25.10.+s Nuclear reactions involving few-nucleon systems

1 Introduction

One of the main questions of η-meson physics concerns the existence of η-nucleus bound states, a possibility predicted in a pioneering work by Haider and Liu [1]. In this context it is worth mentioning that according to [2] the mean free path of η mesons in a nuclear medium is about 2 fm, *i.e.*, less than the size of a typical nucleus. A necessary condition for the existence of η-nuclei, hence, appears to be satisfied. A further indication is the observation of pionic nuclei (deeply bound pion states inside nuclei) [3], which were predicted theoretically in [4].

Recently, in a number of theoretical investigations based on quite different approaches, such as the mean-field approximation [5], the optical model [6], and few-body calculations [7–9], the idea of η-nuclei was given a rather firm ground. Experimentally, after the discouraging attempt of ref. [10], certain evidence for the existence of η-nuclei was noticed by two groups [11,12]. An additional indication is given by the enhancement of near-threshold η production in the reaction $np \to d\eta$ as reported by the Uppsala group [13]. This observation is most likely to be understood as the effect of a near-threshold bound or resonant ηd state. Theoretically, the possibility of forming η-meson bound states with very light nuclei, such as deuteron or ^{4}He, was first indicated in refs. [7] and [14]. Moreover, it was suggested [15] that even quite exotic systems containing an η meson and a hyperon (η-hypernuclei) may exist.

Being of interest by itself, the existence of η-nuclei would also shed new light on various fundamental problems of particle physics. For example, the study of η-nuclei would give a clue for understanding the possible restoration of chiral symmetry in a nuclear medium, or its partial restoration which may occur at normal densities $\sim 0.17\,\mathrm{fm}^{-3}$ [16–18]. A further aspect, which makes such systems interesting for other fields of nuclear physics, is the modification of the two-body ηN force by the nuclear medium. Being enhanced due to the resonant character of the ηN-interaction, general properties of such modifications should become particularly evident in this case. Especially, the structure of the S_{11}-resonance embedded in a nuclear medium may be studied. This would exhibit certain details of the effective Lagrangian models of such a resonance [19], and shed some light, for example, on chiral models which suggest a reduction of its partial width for decaying into the ηN channel [18], or assume S_{11} to be a quasi-bound state of the K-meson and a Σ-hyperon [20]. The interesting suggestion of considering S_{11} as a joint manifestation of threshold (cusp) and resonance phenomena [21] could also be checked.

In a sense, the problem of choosing the most adequate model for describing the S_{11}-resonance is similar to the usual difficulty of choosing the "correct" two-body potential. A reliable judgment on such a choice can be given when this potential (or the resonance) is used in a few-body system where its off-shell properties are exposed. This requires a treatment based on rigorous few-body theory. The Alt-Grassberger-Sandhas (AGS) equations [22], which are employed in this work to calculate the η-deuteron elastic scattering amplitude, belong to this category.

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The Faddeev-type coupling of these equations guarantees uniqueness of their solutions. Moreover, as equations for the elastic and rearrangement operators they are well-defined in momentum space, providing thus the desired scattering amplitudes in a most direct and technically reliable manner. The advantage of working with coupled equations is not only suggested by questions of uniqueness, but also by the relevance of rescattering effects. Indeed, in recent calculations of η-photoproduction from the deuteron it was found that rescattering terms give a significant contribution to the corresponding amplitude [23].

In momentum space the system of AGS equations consists, after partial-wave decomposition, of three coupled two-dimensional integral equations. It is customary to reduce the dimension of these equations by representing the two-body T-operators in separable form,

$$t_\alpha(z) = |\chi_\alpha\rangle\tau_\alpha(z)\langle\chi_\alpha| \ , \qquad \alpha = 1,2,3 \ , \tag{1}$$

where the three values of the subscript correspond to the two-body subsystems NN, ηN_1, and ηN_2. With these separable T-operators the integral AGS equations become one-dimensional (for details see ref. [24]). Moreover, since the nucleons are identical, instead of three we have to solve only two integral equations, viz.

$$\begin{aligned} X_{11}(z) &= 2\langle\chi_1|G_0(z)|\chi_2\rangle\tau_2\left(z-\mathbf{q}_2^2/2M_2\right)X_{21}(z)\\ X_{21}(z) &= \langle\chi_2|G_0(z)|\chi_1\rangle + \langle\chi_2|G_0(z)|\chi_1\rangle\tau_1\\ &\quad\times\left(z-\mathbf{q}_1^2/2M_1\right)X_{11}(z) + \langle\chi_2|G_0(z)|\chi_2\rangle\tau_2\\ &\quad\times\left(z-\mathbf{q}_2^2/2M_2\right)X_{21}(z) \ . \end{aligned} \tag{2}$$

In these equations G_0 is the free resolvent of the three particles involved, $\mathbf{q}_\alpha$ the momentum of particle α relative to the $(\beta\gamma)$ subsystem, M_α the corresponding reduced mass, and z the three-body energy. We denote the η-meson as particle 1 and use the complementary notation to label the two-body T-operator of the $(\beta\gamma)$ pair as t_α. The transition operators $X_{\beta\alpha}$ act exclusively on the relative momentum states $\langle\mathbf{q}_\beta|$ and $|\mathbf{q}_\alpha\rangle$ and their subscripts 1 and 2 indicate the two-fragment partitions (1,23) and (2,31) respectively. Therefore, X_{11} describes the elastic transition $1(23) \to 1(23)$, while X_{21} represents the rearrangement process $1(23) \to 2(13)$.

The S-wave nucleon-nucleon separable potential is adopted from ref. [25] with its parameters slightly modified to be consistent with more recent NN data (see ref. [9]). The η-nucleon T-matrix is taken in the form

$$t_{\eta N}(p',p;z) = (p'^2+\alpha^2)^{-1}\frac{\lambda}{(z-E_0+i\Gamma/2)}(p^2+\alpha^2)^{-1} \ , \tag{3}$$

consisting of two vertex functions and the S_{11}-propagator in between [8]. It corresponds to the process $\eta N \to S_{11} \to \eta N$ which at low energies is dominant. The range parameter $\alpha = 3.316\ \text{fm}^{-1}$ was determined in ref. [26], while E_0 and Γ are the parameters of the S_{11}-resonance [27],

$$E_0 = 1535\,\text{MeV} - (m_N + m_\eta) \ , \qquad \Gamma = 150\,\text{MeV} \ .$$

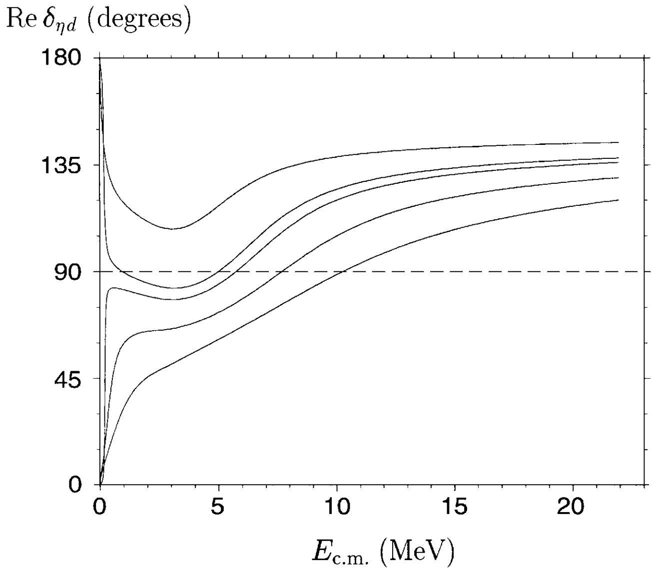

Fig. 1. Real part of the η-deuteron phase-shift as a function of the collision energy. The five curves correspond (starting from the lowest one) to $\text{Re}\, a_{\eta N}$= 0.55 fm, 0.65 fm, 0.725 fm, 0.75 fm, and 0.85 fm.

The strength parameter λ is chosen to reproduce the η-nucleon scattering length $a_{\eta N}$,

$$\lambda = \frac{\alpha^4(E_0 - i\Gamma/2)}{(2\pi)^2\mu_{\eta N}}a_{\eta N} \ . \tag{4}$$

It is customary to use complex $a_{\eta N}$ the imaginary part of which accounts for the flux losses into the πN channel. The value of $a_{\eta N}$ is not accurately known. Different analyses [28] provided for $a_{\eta N}$ values in the range

$$0.27\ \text{fm} \le \text{Re}\, a_{\eta N} \le 0.98\ \text{fm} \ , \tag{5}$$

$$0.19\ \text{fm} \le \text{Im}\, a_{\eta N} \le 0.37\ \text{fm} \ . \tag{6}$$

Recently, however, most of the authors agreed that $\text{Im}\, a_{\eta N}$ is around 0.3 fm. But for $\text{Re}\, a_{\eta N}$ the estimates are still very different (compare, for example, refs. [29] and [30]). Most of the results presented in this paper are, therefore, obtained using $\text{Im}\, a_{\eta N}$=0.3 fm and several values of $\text{Re}\, a_{\eta N}$ within the above interval.

We solved eqs. (2) for ηd collision energies varying from zero (ηd-threshold, $z = E_d$) up to 22 MeV. As is well known (see, for example, [24]), the kernels of these equations, when expressed in momentum representation, have moving logarithmic singularities for $z > 0$. In the numerical procedure, we handle it with the method suggested in ref. [31]. The main idea of this method consists in expanding the unknown solutions (in the area covering the singular points) in certain polynomials and subsequent analytic integration of the singular part of the kernels.

The results of our calculations are presented in figs. 1–4 and in table 1. In fig. 1 the energy dependence of the ηd phase-shifts for five different choices of $\text{Re}\, a_{\eta N}$ is shown, namely, for 0.55 fm, 0.65 fm, 0.725 fm, 0.75 fm, and 0.85 fm. The larger this value, the stronger is the ηN attraction.

Table 1. Energy and width of the ηd-resonance for various choices of $\mathrm{Re}\, a_{\eta N}$.

$\mathrm{Re}\, a_{\eta N}$ (fm)	$E^{\mathrm{res}}_{\eta d}$ (MeV)	$\Gamma_{\eta d}$ (MeV)
0.55	8.24	9.15
0.65	7.46	8.45
0.675	7.14	7.61
0.70	6.79	6.90
0.725	6.41	6.31
0.75	6.01	5.87
0.85	4.39	5.79
0.90	3.73	6.81

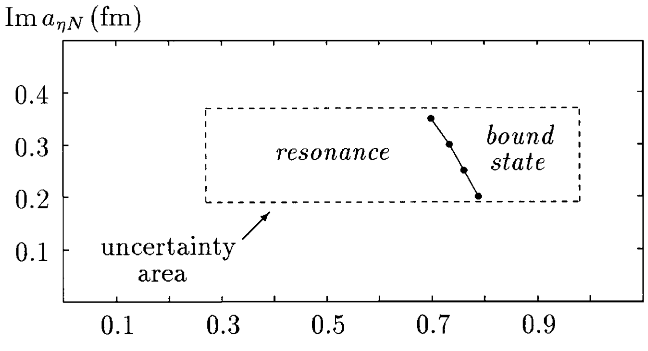

Fig. 2. The uncertainty area (dashed rectangle) given by formulae (5), and the critical values of $a_{\eta N}$ (filled circles) to the right of which the strength of ηN attraction is sufficient for ηd bound-state formation.

The change in the character of these curves, hence, reflects the growth of the attractive force between the η-meson and the nucleon. The lower three curves for $\mathrm{Re}\, \delta_{\eta d}$ corresponding to the smaller values of $\mathrm{Re}\, a_{\eta N}$ start from zero, the two curves corresponding to the strong attraction start from π. According to Levinson's theorem, the phase shift at threshold energy is equal to the number of bound states n times π. We found that the transition from the lower family of the curves to the upper one happens at the critical value $\mathrm{Re}\, a_{\eta N}$=0.733 fm. Therefore, the ηN force, which generates $\mathrm{Im}\, a_{\eta N} = 0.3$ fm and $\mathrm{Re}\, a_{\eta N} > 0.733$ fm, is sufficiently attractive to bind η inside the deuteron.

For three other choices of $\mathrm{Im}\, a_{\eta N}$ within the uncertainty interval, namely, 0.20 fm, 0.25 fm, and 0.35 fm, the corresponding critical values of $\mathrm{Re}\, a_{\eta N}$ turned out to be 0.788 fm, 0.761 fm, and 0.698 fm. In the complex $a_{\eta N}$-plane (see fig. 2) the corresponding points form a curve separating the uncertainty area into two parts. If $a_{\eta N}$ is to the right of this curve, the ηd-system can be bound. This is the first conclusion of our calculations.

The second conclusion concerns the peaks in the energy dependence of the total elastic cross-section (see fig. 3), indicating that a resonance appears in the ηd-system. Of course, not every maximum of the cross-section is a resonance, but the Argand plots, shown in fig. 4, prove that the maxima we found are resonances. Their positions and widths for various choices of $\mathrm{Re}\, a_{\eta N}$ are given in table 1. It should be noted that, while the resonance energy is determined in our calculations exactly (as the maximum of the function $\sin^2 \mathrm{Re}\, \delta_{\eta d}$), the corresponding width is obtained by fitting the cross-section with a Breit-Wigner curve. Therefore, the values of $\Gamma_{\eta d}$ given in table 1 should be considered only as rough estimates.

The presence of a resonance before a quasi-bound state appears is not surprising. With increasing attraction the

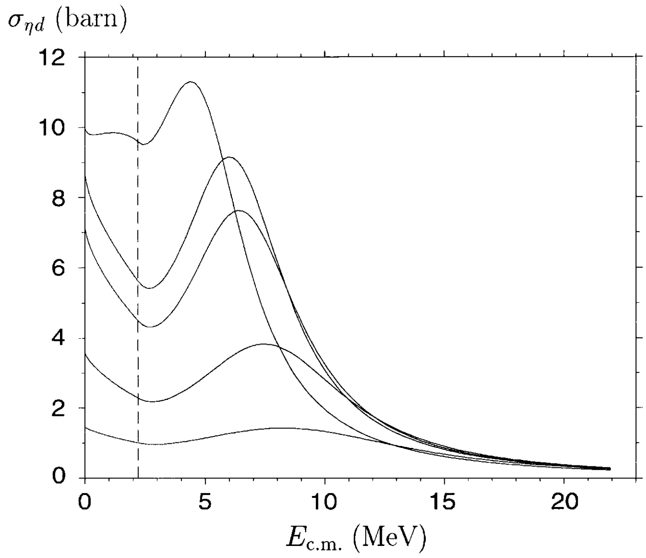

Fig. 3. Total cross-section (integrated over the angles) for elastic η-deuteron scattering as a function of collision energy. The five curves correspond (starting from the lowest one) to $\mathrm{Re}\, a_{\eta N}$= 0.55 fm, 0.65 fm, 0.725 fm, 0.75 fm, and 0.85 fm. The dashed line indicates the deuteron break-up threshold.

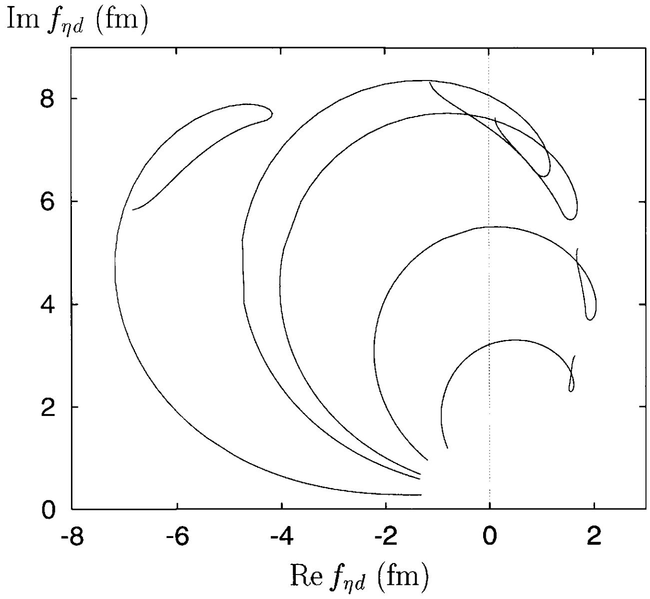

Fig. 4. Argand plot for the ηd elastic scattering amplitude in the energy interval from 0 to 22 MeV. The five curves correspond (from right to left) to $\mathrm{Re}\, a_{\eta N}$= 0.55 fm, 0.65 fm, 0.725 fm, 0.75 fm, and 0.85 fm. When the energy increases the corresponding points move anticlockwise.

poles of the S-matrix should move in the complex plane from the resonance area to the quasi-bound state area. This is exactly what our calculations indicate. In ref.[9] we showed that such transition of the pole happens when $\mathrm{Re}\, a_{\eta N}$ changes from 0.25 fm to 1 fm. Here we found the corresponding critical value of $\mathrm{Re}\, a_{\eta N}$ exactly.

The resonant behaviour of ηd elastic scattering should be seen in various processes involving ηd-system in their final states, such as $\gamma d \to d\eta$ and $np \to d\eta$. Indeed, the corresponding amplitudes $\langle\psi_{\rm out}|\mathcal{O}|\psi_{\rm in}\rangle$ involve the ηd wave function $\psi_{\rm out}$ which, in the vicinity of the resonance, strongly depends on the total energy. Its resonant growth at short distances may enhance the transition probability.

Recent measurements [13] of the η production in the np collisions reveal a bump of the cross-section for the reaction $np \to d\eta$ at a c.m. energy below 5 MeV. If we suppose that the energy dependence of this cross-section is mainly determined by the final-state interaction, then this bump can be explained by the existence of an ηd-resonance. Moreover, since the bump was observed below 5 MeV, the resonance positions given in table 1 imply the rough lower bound $\mathrm{Re}\, a_{\eta N} \geq 0.75$ fm. For a reliable estimate, however, one has to perform an explicit calculation of the corresponding cross-section. A recent analysis [30] of this reaction with a non-resonant final-state interaction is consistent with the data of ref. [13] only if $\mathrm{Re}\, a_{\eta N} = 0.30$ fm. Finally, it is interesting to note that there is tentative evidence for a similar bump in the low-energy region in the reaction $\gamma d \to X\eta$ [32].

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