The search for exotic η-nucleus complexes in light nuclei

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We revisit the long-standing problem of the possible formation of metastable states in collisions of the eta-meson with atomic nuclei. The two-body eta–nucleon interaction is described by a local potential, which is constructed by fitting known low-energy parameters of this interaction. The many-body eta–nucleus potential obtained within the folding model is used to search for metastable states of the systems formed by the eta-meson with hydrogen and helium isotopes. We find that all these systems generate strings of overlapping resonances.

Introduction

The η-meson was discovered in 1961 by a Johns Hopkins University team working with the Bevatron accelerator at Berkeley.[1] This happened when physicists already understood that many 'elementary' particles known at that time were not in fact elementary, and were attempting to learn their compositions. Theorists were looking for an adequate classification of the particles, based on group theory, and experimentalists supplied them with the necessary data. The Berkeley discovery came at the right time, since the η-meson fitted into the octet of other mesons, a group of particles with more or less similar properties, in accordance with the formal group theory classification that eventually evolved into quark theory.

Since its discovery, extensive theoretical and experimental efforts have been devoted to achieving a better understanding of the η-meson's properties and its interaction with other particles. The motivation was fired by the special role played by the η-meson in particle physics. For example, its quark composition is such that it opens up new possibilities for investigating the breakdown of the Okubo–Zweig–Iizuka (OZI) rule[2] and charge-symmetry breaking (CSB).[3] The latter can be attributed to quantum mixing of the quark states corresponding to the η and π^0 mesons.

Although the η-meson is four times heavier, it is in many respects similar to the π^0-meson. Both are neutral, spinless, and have almost the same lifetime, $\sim 10^{-18}$ s. The kinship between the two mesons manifests itself very clearly in their decay modes. They are the only mesons that have a high probability of pure radiative decay. The pion almost entirely (98.798%) decays into the radiative channel $\pi^0 \to \gamma + \gamma$. For the η, purely radiative decay is also the most probable mode,[4]

$$\eta \to \begin{cases} \gamma + \gamma & (38.8\%) \\ \pi^0 + \pi^0 + \pi^0 & (31.9\%) \\ \pi^+ + \pi^- + \pi^0 & (23.6\%) \\ \pi^+ + \pi^- + \gamma & (\ \ 4.9\%) \\ \text{other decays} & (\ \ 0.8\%) \end{cases} .$$

It is believed that the π^0 and η-mesons are related to each other such that their physical quantum states are mixtures of each other,

$$\begin{aligned} |\pi^0\rangle &= |\tilde{\pi}\rangle \cos\theta - |\tilde{\eta}\rangle \sin\theta, \\ |\eta\ \rangle &= |\tilde{\pi}\rangle \sin\theta + |\tilde{\eta}\rangle \cos\theta, \end{aligned}$$

where $|\pi^0\rangle$ and $|\eta\rangle$ are the physically observed particles, whereas $|\tilde{\pi}\rangle$ and $|\tilde{\eta}\rangle$ are their pure isotopic states and the mixing angle is expected to be $0.01 < \theta < 0.02$. Moreover, this mixing is one of the reasons why the charge symmetry is broken — why, for example, the proton and neutron are different (see refs 5–8).

When π^0 and η are viewed as elementary particles, therefore, they look quite similar. However, in their interaction with nucleons, their difference is clearly revealed. First of all, the large difference in mass of the η ($\sim$547.45 MeV) and π^0 ($\sim$135 MeV) mesons should manifest itself in the meson–nucleon dynamics. This is indeed the case at low energies. For example, the S_{11}-resonance $N^*(1535)$ is formed in both $\pi^0 N$ and ηN collisions, but at different energies,

$$\begin{aligned} E^{res}_{\pi N}(S_{11}) &= 1535\ \text{MeV}\ - m_N - m_\pi \ \approx\ 458\ \text{MeV}\ , \\ E^{res}_{\eta N}(S_{11}) &= 1535\ \text{MeV}\ - m_N - m_\eta \ \approx\ 49\ \text{MeV}\ . \end{aligned}$$

As can be seen, this resonance is very close to the ηN-threshold due to the large mass of the η-meson. Furthermore, it is very broad, with $\Gamma \approx 150$ MeV, covering the whole low-energy region of the ηN interaction. As a result, the interaction of nucleons with η-mesons in this region, where the S-wave interaction dominates, is much stronger than with pions.

Another consequence of the S_{11} dominance is that the interaction of the η-meson with a nucleon can be considered as a series of formations and decays of this resonance, as shown in Fig. 1. Independently of the formation channel, the intermediate $N^*(1535)$-resonance decays with almost equal probabilities into the ηN and πN channels. Thus, in the energy region covered by the S_{11}-resonance, the ηN and πN interactions should be treated as a coupled-channel problem. When such an analysis was performed, it was found that the near-threshold ηN interaction is attractive.[9] This is an important feature of the ηN interaction since it raises the question of whether this attraction is strong enough to bind the η-meson inside a nucleus. Such a possibility looked exciting to us and so we began a search for metastable η-nucleus complexes.

Since η-mesons decay very quickly, it is impossible to produce beams of them and indeed they can be observed only in the final stages of certain nuclear reactions. This makes the investigation of η-meson dynamics quite complicated. If an η-meson could be sustained inside a nucleus for some time, however, it would expose itself for a relatively long period in a series of successive

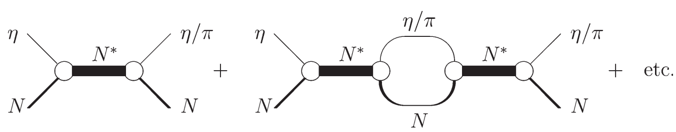

Fig. 1. Schematic representation of the resonant two-channel scattering $\eta N \to N^* \to \eta N$ and $\eta N \to N^* \to \pi N$. The notation η/π means η or π.

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interactions with nucleons; that is, inside the nucleus it would undergo a series of absorptions and emissions through formation and decay of the $N^*(1535)$-resonance, as depicted in Fig. 2.

In such a series of interactions, after each decay of the S_{11}-resonance the η-meson is generated anew. The lifetime of an η-mesic nucleus would not be limited therefore by the lifetime of the meson itself. However, such an η-nucleus complex cannot be stable, since eventually the $N^*(1535)$-resonance will produce a pion with a large kinetic energy of ~400 MeV (thanks to its small mass), which will enable it to escape. It is therefore clear that if an η-meson is bound inside a nucleus, it can only be in a quasi-bound (metastable) state with a non-zero width.

Being of interest in themselves, the existence of η-nuclei would also shed new light on various fundamental problems of particle physics. This is why so much effort has been devoted to understanding η-nucleus dynamics and searching for long-lived η-nucleus complexes. Our work is another step in this direction.

In contrast to all previous calculations, which were based on a one-term separable ηN interaction, we construct a local ηN potential. This enables us to use the powerful Jost function method to look for metastable states formed by the η-meson with hydrogen and helium isotopes. We found that all these systems generate strings of overlapping resonances.

Quest for quasi-bound η-nucleus systems

A first estimation obtained in terms of optical potential theory, put a lower bound on the number A of the nucleons that would be sufficient to bind the η-meson, namely, $A \geq 12$ (ref. 10). Thereafter, other theoretical investigations were devoted to this problem. All of them predicted η-nucleus bound states obeying the same constraint of $A \geq 12$. However, the first experimental attempt to find η-nucleus bound states with lithium, carbon, oxygen, and aluminium produced negative results.[11] Even so, this experimental outcome did not discourage theoreticians from examining the possibility of η-nucleus binding.

The relatively large scattering lengths obtained for the η^3He and η^4He systems cast some doubt on the $A \geq 12$ constraint.[12] Moreover, in the measurements of η-meson production on nuclei by γ-quanta and other particles, it was found that the cross-section depends strongly on the energy and is practically isotropic (see, for example, refs 13, 14), which can be explained by the formation of either a bound or a resonant η-nucleus state. Indeed, if the η-meson is trapped by a nucleus, the bound or resonant state can be formed at a specific energy, any shift from which must lead to a significant decrease of the cross-section, which means strong energy dependence. Moreover, after being captured, the meson 'forgets' the direction of incidence, which means that the decay of such a state must be isotropic.

The first microscopic few-body calculations of η-meson scattering from d, t, ^{3}He, and ^{4}He nuclei are presented in refs 15 and 16. By locating the poles of the S-matrix in the complex momentum plane, we found that with the uncertainties of the parameters of the η-nucleon potential, even the existence of an η-deuteron quasi-bound state could not be excluded. This work stimulated interest in possible bound states of the η-meson with light nuclei. Theoretical papers claimed the discovery of such states on the basis of simplified calculations. Recent experimental work (for a review, see ref. 17) also gave strong evidence supporting the existence of near-threshold bound or resonance states of the η with ^{11}B, ^{11}C, ^{4}He, and even deuteron. This evidence, however, remains inconclusive because they are all based on indirect observations of the enhancement of the final state interaction between the η and the nucleus, which takes place at low positive energies. The quasi-bound states, if any, are also not far from the zero energy, but on the negative side. In this paper, we explore the possibility of states with near-threshold positive energies.

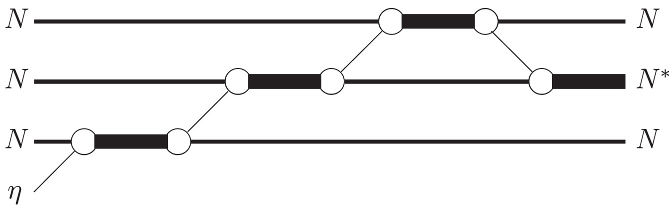

Fig. 2. Schematic representation of the resonant η-nucleus scattering.

η-nucleon potential

As mentioned in the introduction, when the ηN interaction is considered, it is necessary to take into account the πN channel, i.e. the transitions $\eta N \to \pi N \to \cdots$. The fact that the pion has a much smaller mass offers a way to simplify the treatment of this two-channel problem. Indeed, the transition $\eta N \to \pi N$ is accompanied by the release of ~400 MeV of kinetic energy. As a result, the pion and nucleon have a very short time to interact. This means that the probability of returning to the initial channel via the reverse transition $\pi N \to \eta N$ is negligible. We can therefore safely assume that if the η-meson happens to undergo the transformation into the pion, it never reverts and is lost forever.

In quantum mechanics, such disappearances of particles are formally described by adding an imaginary part to the potential. Thus, instead of considering the ηN and πN systems as a two-channel problem, we can assume that the ηN interaction is described by an effective one-channel complex potential.

Since η beams are not available and direct scattering experiments with them are not possible, the quantitative data that can be used to construct the ηN potential are scarce. The only known quantities are the position $E_{S_{11}}$ of the pole of the ηN scattering amplitude, corresponding to the S_{11} resonance, and the scattering length a_{η_N}, i.e. the value of the amplitude at zero collision energy. Even this scarce information,[18]

$$E_{S_{11}} \approx (49 - i75)\,\mathrm{MeV}\ , \tag{1}$$

$$0.2\,\mathrm{fm} \leq \mathrm{Re}\, a_{\eta N} \leq 1.0\,\mathrm{fm}\ , \tag{2}$$

$$0.2\,\mathrm{fm} \leq \mathrm{Im}\, a_{\eta N} \leq 0.4\,\mathrm{fm}\ , \tag{3}$$

is not very accurate. In previous publications, the ηN potential was constructed in momentum space and in a separable form, i.e. it was non-local. Here we present the first attempt to describe the ηN interaction by a local potential in configuration space. Although it is more difficult to construct such a potential, it opens up new possibilities for exploring the resonance spectra of η-nucleus systems. This is because with a local potential, we can use the Jost function method for locating resonances.

As a starting point, we chose the following functional form for the ηN potential

$$V_{\eta N}(r) = a_1 e^{-b_1(r_1 - r)^2} - a_2 e^{-b_2 r^2} - i a_3 e^{-b_3 r^2}\ , \tag{4}$$

where the first term is a barrier responsible for the S_{11} resonance, the second term represents a short-range attraction, and the last one is the absorptive part that takes care of all inelastic processes.

In order to find appropriate values for the parameters of the potential given by Equation (4), we used the following fitting procedure. For any choice of the parameters, we can locate the ηN resonance as well as calculate the ηN scattering length using the Jost function method described in the section 'Method for locating resonances' below. Thus the calculated resonance energy $E_{S_{11}}$ and the scattering length a_{η_N} are therefore functions of the parameters $\{r_1, a_1, a_2, a_3, b_1, b_2, b_3\}$. The optimal parameters were determined by minimizing the difference function

$$F(r_1, a_1, a_2, a_3, b_1, b_2, b_3) = \left|\tilde{E} - E_{S_{11}}\right|^2 + \left|\tilde{a}_{\eta N} - a_{\eta N}\right|^2 \ , \tag{5}$$

with the goal value for the resonance energy

$$\tilde{E} = (49 - i75)\,\mathrm{MeV}$$

and for the scattering length

$$\tilde{a}_{\eta N} = (0.75 + i0.27)\,\mathrm{fm} \ ,$$

which is considered as the most probable value within the uncertainty intervals,[2,3] since different analyses give the results concentrated around it. The minimum of the difference function was found with the set of parameters shown in Table 1, which give

$$E_{S_{11}} = (48.57 - i75.05)\,\mathrm{MeV}$$

and

$$a_{\eta N} = (0.75 + i0.27)\,\mathrm{fm} \ .$$

In the table, the parameters are given with all the decimal places obtained in the calculations, even though only approximate data are available (refs 1–3). The reason for this is that, first, in the fitting procedure the chosen goal values $\tilde{E}$ and $a_{\eta N}$ considered to be exact, and, second, the global minimum of the function (5) is very sharp, i.e. sensitive to small variations of the parameters. So, by keeping their values exactly as they were produced by computer, we insure ourselves against a deviation from the minimum that may be caused by some combinations of the deviations in the parameter values.

Although there are seven parameters and only four numbers to fit, the choice of the parameters is unique. This is because we found the global minimum of Equation (5), which is unique. The potential corresponding to these parameters is shown in Figs 3 and 4.

The two-body ηN potential is the main building block of our model. Using it, we construct the effective potentials that describe the interaction of the η-meson with light nuclei. This is done in the framework of the folding model.

η-nucleus folding potential

When the η-meson approaches a nucleus, it experiences the forces generated by all the nucleons within the nucleus. The meson moves relative to the nucleus and the nucleons move inside the nucleus. At every instance, the total η-nucleus potential energy is a sum of individual ηN_i potentials, depending on their individual positions. The η-nucleus potential is therefore a complicated function of time. This can be significantly simplified, however, if we take into account the fact that at low collision energies (which we are considering) the nucleons move inside the nucleus much faster than the meson approaches it or moves away. This means that while the meson travels a small distance, the nucleons have enough time to adopt all possible spatial configurations, i.e. the meson experiences the collective force which is the statistical average over all possible configurations of the nucleons. In other words, it moves in the potential field described by the effective folding potential

$$V_{\eta A}(r) = \int V_{\eta N}(|\vec{r} + \vec{r'}|)\rho_A(\vec{r'})\mathrm{d}\vec{r'} \ , \tag{6}$$

where vector $\vec{r}$ points from the meson to the centre of mass of the nuclues, and ρ_A is the nuclear density determined by its wave function.

Table 1. Parameters of the ηN potential.[4]

Parameter	Value
r_1	1.95616478619031975 fm
a_1	57.5826586837329657 MeV
a_2	26.8157044304329091 MeV
a_3	0.603932024464326478 MeV
b_1	0.0715471865601824408 fm^{-2}
b_2	0.0271505486074286040 fm^{-2}
b_3	0.0338015704618582769 fm^{-2}

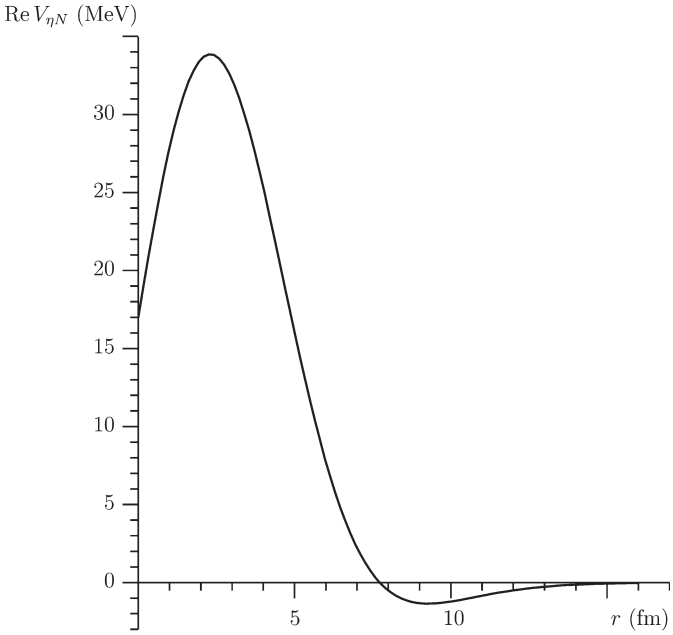

Fig. 3. Real part of the ηN potential (4) as a function of the distance between the η-meson and *N*.

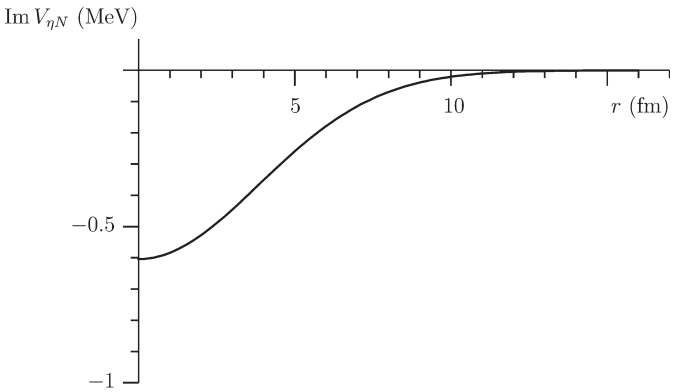

Fig. 4. Imaginary part of the ηN potential (4) as a function of the distance between the η-meson and *N*.

Method for locating resonances

Having constructed the effective η-nucleus potential, we reduced the η-nucleus problem to an effective two-body problem which can be solved using the Jost function method developed in refs 19–23. In this method, the η-nucleus wave function $\psi_\ell(k, r)$ obeying the radial Schrödinger equation

$$\left[\partial_r^2 + k^2 - \ell(\ell+1)/r^2\right]\psi_\ell(k,r) = U_{\eta A}(r)\psi_\ell(k,r) \ , \tag{7}$$

where

$$U_{\eta A}(r) = \frac{2\mu_s}{\hbar^2} V_{\eta A}(r) \tag{8}$$

and $k = \sqrt{2\mu_s E/\hbar^2}$ is the η-nucleus relative momentum, is sought in the form

$$\psi_\ell(k,r) = h_\ell^{(-)}(kr) F_\ell^{(\text{in})}(k,r) + h_\ell^{(+)}(kr) F_\ell^{(\text{out})}(k,r) \ . \tag{9}$$

In this ansatz, the incoming and outgoing waves $h_\ell^{(-)}(kr)$ and $h_\ell^{(+)}(k)$ are embedded explicitly. Two new unknown functions $F_\ell^{(\text{in})}(k,r)$ and $F_\ell^{(\text{out})}(k,r)$ obey the system of coupled first-order equations

$$\begin{cases} \dfrac{d}{dr} F_\ell^{(\text{in})} &= -\dfrac{1}{2ik} h_\ell^{(+)} U_{\eta A} \left[h_\ell^{(-)} F_\ell^{(\text{in})} + h_\ell^{(+)} F_\ell^{(\text{out})} \right] \ , \\ \dfrac{d}{dr} F_\ell^{(\text{out})} &= \dfrac{1}{2ik} h_\ell^{(-)} U_{\eta A} \left[h_\ell^{(-)} F_\ell^{(\text{in})} + h_\ell^{(+)} F_\ell^{(\text{out})} \right] \ , \end{cases} \tag{10}$$

with the boundary conditions

$$\lim_{r\to 0} F_\ell^{(\text{in/out})}(k,r) = 1 \ . \tag{11}$$

This system is equivalent to the initial Schrödinger equation (7) but is more convenient for solving the resonance problem.

The differential equations (10) can be solved numerically from the origin to a sufficiently distant point $r = R$, where the potential vanishes (causing the right-hand sides of the equations to disappear) and therefore $F_\ell^{(\text{in/out})}(k,r)$ become constant. These constants

$$f_\ell^{(\text{in})}(k) = \lim_{r\to\infty} F_\ell^{(\text{in})}(k,r) \tag{12}$$

and

$$f_\ell^{(\text{out})}(k) = \lim_{r\to\infty} F_\ell^{(\text{out})}(k,r) \tag{13}$$

are the Jost functions that determine asymptotic behaviour of the wave function

$$\psi_\ell(k,r) \underset{r\to\infty}{\longrightarrow} h_\ell^{(-)}(kr) f_\ell^{(\text{in})}(k) + h_\ell^{(+)}(kr) f_\ell^{(\text{out})}(k) \ . \tag{14}$$

Resonances are such points in the fourth quadrant of the complex momentum plane, at which the physical wave function (9) has only outgoing waves in its asymptotic behaviour.[14] This means that the resonances can be located as zeroes of the Jost function,

$$f_\ell^{(\text{in})}(k) = 0 \ . \tag{15}$$

For a complex root of this equation, the resonance energy

$$E = E_\text{r} - \frac{i}{2}\Gamma \tag{16}$$

has a negative imaginary part, which is called the resonance width. The width Γ determines how long the resonance lives.

η-nucleus resonances

In this paper, we present the results obtained for the four lightest nuclear isotopes, namely, deuteron (d), triton (t), ^{3}He (τ), and ^{4}He (α). The density functions $\rho_A(r)$ for these nuclei, which are needed for constructing the folding potential (6), were calculated using symmetric wave functions given in ref. 15.

The low-energy ηN and η-nucleus interaction is dominated by the S_{11} resonance, which means that all higher partial waves make a negligible contribution as compared to the one coming from the S-wave. In all our calculations, we therefore assume that $\ell = 0$.

In order to locate possible η-nucleus resonances, we searched for zeroes of the Jost function $f_\ell^{(\text{in})}(k)$ in the fourth quadrant of the complex k-plane. The search was done using Newton's method.[24] For each point in the k-plane, the Jost function was obtained by solving the differential equations (10) from $r = 0$ to a point sufficiently far from the origin where the potential vanishes and the function $F_\ell^{(\text{in})}(k,r)$ reaches its limit value.[12] To avoid the divergence, the integration of the differential equations was performed along the line $r = |r|\exp(i\theta)$ in the complex r-plane with the rotation angle θ such that Im kr remains positive (the reasons for this are given in refs 19–23).

The resonance points thus found are given in Tables 2–5. Their distribution in the energy plane is shown in Fig. 5. For all isotopes considered, the distribution of the Jost function zeroes in the energy plane follows almost vertical lines. This means that the resonances completely overlap each other. Since the width Γ increases very quickly with the resonance number, only the first resonance for each nucleus may be discerned in the scattering or final state interaction picture. All higher resonances form a collective background.

The resonances for t and τ systems are very close to each other. This is because the number of nucleons in these nuclei is the same. The only difference between them comes from the slight difference in the mean square radius. This radius determines the nuclear density distribution. The mass difference, however, has a negligible effect. Compared with the other nuclei (that is, with deuteron and ^{4}He), we see that the mass number A does make a difference.

Table 2. η–d resonances.

Re k (fm^{-1})	Im k (fm^{-1})	E_r (MeV)	Γ (MeV)
0.516676386	–0.250053354	9.39	23.74
0.557757739	–0.466126622	4.31	47.78
0.749837117	–0.581803575	10.28	80.17
0.850332662	–0.734765811	8.42	114.81
0.944632738	–0.816615407	10.36	141.75
1.051964840	–0.926330329	11.42	179.06
1.126248400	–1.014329900	11.01	209.92
1.234325320	–1.111359490	13.25	252.08

Table 3. η–t resonances.

Re k (fm^{-1})	Im k (fm^{-1})	E_r (MeV)	Γ (MeV)
0.909378894	–0.219888233	33.09	33.99
1.081057020	–0.603766038	34.17	110.94
1.237657840	–0.927171221	28.56	195.04
1.348924640	–1.079850840	27.77	247.58

Table 4. η–τ resonances.

Re k (fm^{-1})	Im k (fm^{-1})	E_r (MeV)	Γ (MeV)
0.891738122	–0.226294155	31.61	34.30
1.059055894	–0.604922991	32.11	108.90
1.213812711	–0.917415813	26.84	189.28
1.330761160	–1.066909850	26.88	241.32

Table 5. η–α resonances.

Re k (fm^{-1})	Im k (fm^{-1})	E_r (MeV)	Γ (MeV)
1.23264042	–0.198489492	60.36	39.92
1.40505908	–0.571114105	67.21	130.92
1.58269994	–0.951647370	65.23	245.72

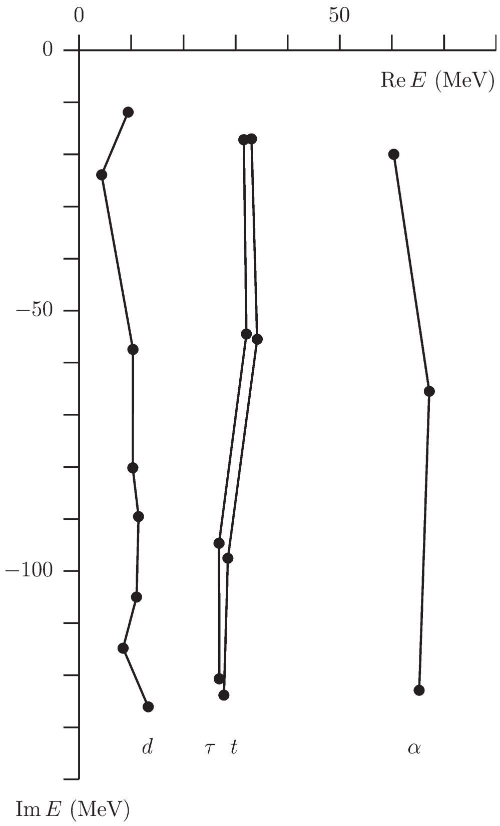

Fig. 5. Resonance points (Jost function zeroes) for the systems formed by the η-meson with four nuclear isotopes (d, t, τ and α).

No quasi-bound state has yet been discovered by experiment.[18] Meanwhile, there are many different predictions based on scattering length calculations, which have serious drawbacks, however, and so are inadequate.[25] The results obtained in this work are based on a rigorous Jost function method and therefore are more reliable. Of course, our predictions cannot be considered as exact and final because they were obtained with an ηN potential constructed using limited experimental data. A more precise potential would slightly shift the resonance positions, but the general picture would remain intact.

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